Illinois/Missouri Applied Harmonic Analysis Seminar
Spring 2008
Open Problems

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Cocktail Party Problem (Maurice Givens Maurice.Givens@gastechnology.org)

Given a signal $f(t) = \sum_{n=1}^{N} \phi_n(t)$ where the "voices" ϕ_n are correlated, one wants to find each individual ϕ_n. The problem is also known as "blind source separation". Sometimes one has additional structure or data, such as frequency separation of the signals, or video data.

Wavelets for other surfaces (Edward Wilson enwilson@math.wustl.edu)

One wants wavelet constructions for other surfaces (e.g., sphere, torus, cylinder). Data on the sphere arises naturally in astronomy. Data on other surfaces arises in computer graphics (e.g., texture maps on objects such as buildings, chairs).

Want explicit constructions, not just abstract results. The work should be useful for engineers

For the sphere, see the recent special issues of the Journal of Fourier Analysis and Applications (Volume 13, Numbers 4 and 6, 2007).

Multiclass Support Vector Machines (Maurice Givens Maurice.Givens@gastechnology.org)

Traditional SVMs separate data into two classes. *Multiclass* separations would be useful.

Related topics: nonlinear regression, recent work of Coifman and Maggioni, reduction of dimension, work of Yi Ma (UIUC).

Mixed Domain Signals (Minh Do minhdo@uiuc.edu)

Consider signals with form $f(x, y, t)$, where x and y are spatial dimensions, and t is a temporal/spectral dimension, where the signal has different structure along the different types of dimensions. One wants to do signal analysis.

Examples: video (joint spatial-temporal processing); spectroscopic imaging (joint spatial-spectral processing). See: A. Jerri (SUNY Potsdam): 1979 Proceedings IEEE.

Compressed Sensing (Justin Haldar haldar@uiuc.edu)

Compressed sensing can allow one to recover sparse/compressible vectors from very limited measurements if the measurement operator satisfies certain properties. One wants:

1. Theorems for complex space $\mathbb{C}^d$ (current theorems are for $\mathbb{R}^d$)
2. Deterministic constructions of good measurement operators

3. Further development of randomized (probabilistic) schemes

4. Computable/tighter performance bounds

5. Fast algorithms

References: `http://www.dsp.ece.rice.edu/cs/`

Limitations of L_2 (Jont Allen `jontalle@uiuc.edu`)

Consider functions

$$\tilde{u}(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}, & t = 0 \\ 1, & t > 0 \end{cases} \tag{1}$$

and

$$u(t) = \begin{cases} 0, & t < 0 \\ \text{undefined}, & t = 0 \\ 1, & t > 0 \end{cases} \tag{2}$$

These two functions are equivalent in L_2, but the differences between them can be important. L_2 doesn't capture all interesting features of functions.

Convergence of Fourier series (Guido Weiss `guido@math.wustl.edu`)

Let E be a subset of $[0,1]$ with positive Lebesgue measure. The Fourier series of the indicator (characteristic) function $\mathbb{1}_E$ converges a.e., but the partial sums might not be uniformly bounded in $L^\infty[0,1]$.

Does there exist a subset $F \subset E$ of positive measure such that the partial sums of $\mathbb{1}_F$ form a bounded sequence in $L^\infty[0,1]$? That is, does a subset F and a constant C exist such that $|\sum_{|n|\le N}\langle \mathbb{1}_F, e_n\rangle e_n(x)| \le C$ for all $x \in [0,1]$ and all $N \ge 0$? (Here $e_n(x) = e^{2\pi i n x}$.)

Volterra series (Maurice Givens `Maurice.Givens@gastechnology.org`)

Problem: estimation of the Volterra series. The series has indeed been estimated, but one would like estimates on the kernel where only $Nu(t)$ and $u(t-\tau_1), \ldots, u(t-\tau_n)$ are known, in the Volterra series defined by

$$Nu(t) = h_0 + \sum_{n=1}^{\infty} \int \cdots \int h_n(\tau_1, \cdots, \tau_n) u(t-\tau_1) \cdots u(t-\tau_n)\, d\tau_1 \cdots d\tau_n.$$

References: work of S. Boyd, Wikipedia article on Volterra series, Wiener's little red book