Holograms
including CGH v. 2.0 (towards iterative algorithms)

Extra equipment needed:

<table>
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<tr>
<th>Magic Cube® Functional Holographic Keyboard</th>
<th>PASCO Transmission Hologram (slide 9115)</th>
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<tr>
<td>Diffractive Optical Elements</td>
<td>PASCO Diffuser</td>
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<tr>
<td>Physical objects to make holograms of (Toys!)</td>
<td>PASCO lenses for magnetic optical rail mounts</td>
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<tr>
<td>LitiHolo kits</td>
<td>PASCO magnetic rail mounts (long and short)</td>
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<tr>
<td>LitiHolo safe lights!</td>
<td>PASCO magnetic optical rail</td>
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<td>Large glass cutter</td>
<td>PASCO turntable for magnetic optical rail</td>
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**Part 1: Functional Holographic Keyboard** - to be shown/discussed *in class*

In Introductory-level Physics coursework, we introduced diffracting objects, such as slits or circular apertures, comprised solely of regions that are either completely opaque or completely transparent. Optical elements of such design are “binary” amplitude modulators. While your eye, on its own, is only sensitive to amplitude modulation, you’ve seen that *phase* profiles can be used to determine the locations of constructive and destructive interference in a (“far field”) diffraction pattern. An element that is designed to control take advantage of this is typically called either a *kinoform* or a *hologram*. In this class, we will use the term “hologram” to refer to any optical element which encodes phase (rather than just amplitude) information. — A good deal of information can be encoded in this sort of (simple to produce) device, as you will see.

![Image](image.jpg)

Fig. 1. **Left**: a diode laser costing much less than a dollar can project a *virtual* keyboard amplitude pattern. **Middle**: the laser passes through a plastic stamped phase-only modulator. **Right**: a second (infra-red) projector illuminates the plane above the table surface to detect finger crossing.
Part 2: Diffractive Optics - to be done in lab

Once you realize that the output of a laser beam can be “re-shaped” by any time lags introduced into the beam path, an easy opportunity for creative expression is unleashed. – At each station there is a plastic sheet of modulo $2\pi$, phase-only “Diffractive Optical Elements” (DOE, from Digital Optics Corporation, founded by Michael Feldman, which he then sold for $59.5 million) containing a variety of textures for you to inspect. Because the index of refraction of plastic is different from that of air, simply stamping a texture into plastic will introduce phase shifts of one part of the beam relative to another. While it may be that a custom DOE is expensive, once a master is made (e.g., in a hard material like glass), it can be used to mold many embossed plastic copies, and so for a mass-market application the cost of each of those can be less than a penny.

The DOE available for you to study today are:

- Diffractive Lens, a.k.a. Fresnel Zone Lens (60-mm focal length, at some unspecified wavelength)
- A “Beam Shaping Element”
- Lenslet Arrays (6 × 6 and 12 × 12 micro-lens arrays) — When might these be useful?
- Line Generators
- Spot Array Generators (1 × 13, 7 × 11, and 8 × 8 arrays)
- Diffusers, a.k.a. Homogenizers / Random Phase Plates (over Dipole, Tripole, Quadrupole, 10-Pole zones)
- Pattern Generator, a.k.a. Beam Splitter / Fan-out-element (IR reference for focusing cell phone cameras)
- Frame Generator, a.k.a. “Grocery check-out aiming guide” for scanning Universal Price Codes
- Linear Gratings (periods of 1.6 µm, 1.2µm)
- Computer-Generated Holograms (CGH):
  - “Micro-Optic” in German, Chinese, Hebrew, Japanese, and Company Logo in English
  - “Eye”
  - “Bride”
  - “Keyboard”

**Step A:** On the PASCO rail system, construct a beam expander using two lenses separated by the sum of their focal lengths. **Which lens should be positioned closer to the laser?** [A quick ray-optics sketch can help you to decide.] — You want the laser beam to be expanded so as to sample the full texture stamped on an individual square of the plastic sheet. Test that the output is “roughly collimated” simply by using an index card: if the separation between lenses is too much, then the beam will shrink as you move the card further away; if the separation is too little, then the beam will expand.

**Step B:** Using your expanded your laser beam, inspect the output generated by the different squares on the transparent plastic sheet.

**Step C:** Select one of the most detailed Computer-Generated Holograms. Is there a detectable loss of image quality if you remove your beam expansion lenses, and use an unexpanded beam (thereby only illuminating a small zone within the plastic square)?

**Step D:** Using a microscope, directly inspect the texture stamped onto one of the simpler Computer-Generated Hologram (i.e., the “Keyboard” image, or the company logo, or “Micro-Optic” in any language), and compare the “direct image” of the stamped texture (seen under a microscope) to your expectations for and understanding of the diffraction pattern produced by this particular plastic square.
Part 3: Holograms produced by Physical Objects (LitiHolo kits)

Step A: Watch this brief video, showing step-by-step instructions for the LitiHolo kits:

![Hologram Kit Quickstart Guide](image)

Step B: With your lab partner, make one single hologram. Examine the result.

Step C: Assuming that you would each like to have your hologram to keep, use the large glass cutter to divide your completed hologram into two halves. With the wisdom of King Solomon, discuss the result in your lab notebook.

Part 4: Holographic Chess (Up)Sets

* CAUTION *
AN OBSERVER SHOULD NEVER PUT THEIR EYE DIRECTLY IN LINE WITH THE LASER BEAM.
USE CARE!

The PASCO Transmission Hologram (slide 9115) is not designed to be projected onto a screen, but instead produces a virtual image. While viewing is optimal if your eye is in the plane defined by the optic axis and the normal to the hologram surface, you must never move your eye onto the optic axis! Replace the component carrier holding your DOE with the rotation table and small component carrier, with the hologram mounted. Turn the rotation table so that the light strikes the hologram at an angle of around 30°. Position a diffuser so that your hologram is fully illuminated, with as much brightness as possible.

Note: If you are unable to find the holographic image, you might try removing all lenses, using the diffuser alone (again, positioned so that the hologram is fully illuminated).
Once you *can* clearly see the image of a chess piece, **keep your viewing angle fixed** and turn the hologram slide upside down, so as to view the chess set “upside down” – and describe, in your lab notebook, any changes you observe in the image. (Whatever you see, you say what you saw.)

Next, **turn the hologram around, so that the front-side becomes the backside** – and note, in your lab notebook, any changes you observe in the image.

**HOW ARE THESE OBSERVATIONS POSSIBLE?!?!??!?!**

Finally, re-orient your hologram (in terms of which side is up and which side is front) to as to yield the strongest image – and then turn the rotation table so that the laser hits the hologram at normal incidence. While **keeping your open eye *OFF* the optic axis**, see if you can find both the \( m = 1 \) and the \( m = -1 \) diffraction orders. Note, in your lab notebook, what you observe. Does these new observations help you to explain any other observations you’ve made?

**Post-lab** considerations:

So far, the only design approach we’ve discussed is non-iterative: a superposition (modulo \( 2\pi \)) of gratings and lenses, to accumulate a series of bright spots in 3D. This “Gratings & Lenses” approach is conceptually simple (and therefore computational fast), but actually quite limited in terms of the ultimate image quality produced, as illustrated in Fig. 2 below:

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Uniformity</th>
<th>Efficiency</th>
<th>Computational Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superposition of “Gratings+Lenses”</td>
<td>0.01</td>
<td>0.29</td>
<td>Fastest</td>
</tr>
<tr>
<td>Method B</td>
<td>0.60</td>
<td>0.94</td>
<td>Fairly Fast</td>
</tr>
<tr>
<td>Method C</td>
<td>0.79</td>
<td>0.93</td>
<td>Fairly Fast</td>
</tr>
<tr>
<td>Method D</td>
<td>1.00</td>
<td>0.68</td>
<td>Very Slow</td>
</tr>
</tbody>
</table>

Fig. 2. Algorithm Tests (case numbers shown are for a target \( 10 \times 10 \) spot array output pattern). Efficiency is a measure of the percent of light that is going where you want it to go. In our lab, we’ve often made use of “Method C” above, as a compromise solution, offering reasonable uniformity without taking too long.
The downfall of simply using a superposition of “Gratings & Lenses” is that, particularly for highly symmetric patterns, such as our benchmark test case, there are spots of unintended constructive interference (“ghost orders”), as well as points of interest that see unintended cancellations. Recall, in exploring the app iHologram, we noted that, generally, when as few as three different linear phase profiles (“blazes”) are superimposed, there will be “singular points,” where all phases meet, which, by symmetry, must be perfectly dark.

In addition to these unintended ways in which energy is redirected, common manufacturing methods in use today tend to yield devices characterized by a discrete number of phase levels, and there are also some undesired discretization effects (e.g., aliasing).

In Part (a) of Fig. 3, phase wrapping has been used. In the limit of a perfectly calibrated device with an infinite number of addressable phase levels and 100% filling factor, the sudden steps from 0 to $2\pi$ would not constitute discontinuities, and the emerging beam would be described by a single wave vector, $\vec{k}$. However, for any real device, the resulting discontinuities mean that the pattern can only be described as a superposition of many different spatial frequencies, corresponding to multiple diffractive orders in the output.

The efficiency of a Diffractive Optical Element is a measure of how much of the light is going where you want it to go. Even the phase profile shown in Part (c) of Fig. 3 succeeds in sending some energy towards the desired location, but since this pattern contains nothing breaking left-right symmetry, it obviously must send just as much energy into the opposite angle (i.e., the $m = -1$ diffraction order) — and so its efficiency clearly could never exceed 50% (and when you consider that some energy goes into higher diffraction orders, it becomes clear that a binary phase pattern has an efficiency that is even further reduced).
The earliest Holographic Optical Traps were created using phase profiles of this sort, by etching calculated patterns into glass DOE. By performing $N$ distinct etch steps, each using a distinct patterned coating (an “etch mask”) to define which regions are/are not exposed to the etchant, it is possible to create $2^N$ distinct phase levels. But because of the challenges associated with carefully aligning $N$ distinct etch masks, it was not uncommon to use binary phase profiles (where $N = 1$). The sacrifice in efficiency is acceptable if you have plenty of laser light to spare.

Once SLMs became available, the optical trapping community largely stopped etching glass, in favor of these dynamically addressable devices. Even with the SLM, though, any filling fraction less than 100% yields “diffraction noise” and, in addition, the finite number of addressable phase levels (typically $2^8 = 256$), coupled with the finite pitch, $D$, of the device, limits the maximum phase gradient that can be physically realized without aliasing. (Quiz? Who said anything about a quiz? What Quiz?)

For real physical devices, the Algorithm Tests shown in Fig. 2 show that a simple superposition of “Gratings & Lenses” tends to suffer from both poor uniformity (in the relative brightness of the target $10 \times 10$ output array of spots) and poor efficiency due to aliasing of large gradients created by the superposition along with unintended dark spots at singular points, unintended bright spots at ghost orders/higher diffraction orders. Hopefully those issues make sense to you.

**Homework:**

In your notebook, discuss what kind of algorithm (or “approach”) you might, in principle, be able to utilize for designing the appropriate texture to stamp into the Diffractive Optical Element, in order to produce the desired (“far field”) diffraction pattern for, say, the Keyboard. — Here, I’m not asking you to actually create the code, but just to describe, in fairly broad terms, something else that could be tried, as an alternative to the superposition of “Gratings & Lenses,” which, while still a good choice for some projects, leaves us seeking other options.

You may gain a few ideas if you try to analyze the direct image of a stamped texture, taken by using a microscope to examine one square of your plastic DOE.

For what it is worth, I will note that the “mystery algorithms” listed in Fig. 2 as Method B, Method C, and Method D were all iterative algorithms, which attempted to minimize some (made-up) metric characterizing key aspects in the difference between what we wanted to see in the output, and whatever a particular iteration of the calculated phase profile might yield. Making up an appropriate metric is what, later in the course, we will call “The Optimization Conundrum” (i.e., be careful what you ask for, as you just might get it!).