Name: ______________________________
Partner(s): ______________________________
Date: ______________________________

Imprinting Phase Profiles

Simple phase profiles imposed by an SLM can redirect beams and/or create *multiple* focal spots, which, if aberration is minimal, can act as Optical Traps for micromanipulation of micro- and nano-components, or cold atoms, creating opportunities for studies of *new physics*.

Example 1: Redirecting a beam

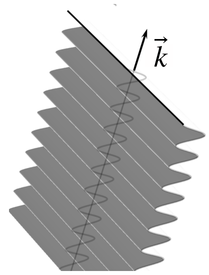

Fig. 1. (from Ch2 of *Optics 2f2*) In a planar wave the phase is *uniform* in a plane orthogonal to the propagation direction.

Consider a plane wave, arriving *at an angle* to a lens, as at bottom left of Fig. 2 below. This *tilt* means different points on a *crest* do not arrive at the lens at the same time. That is, this method of redirecting the focal spot is equivalent to a *phase* retardation.

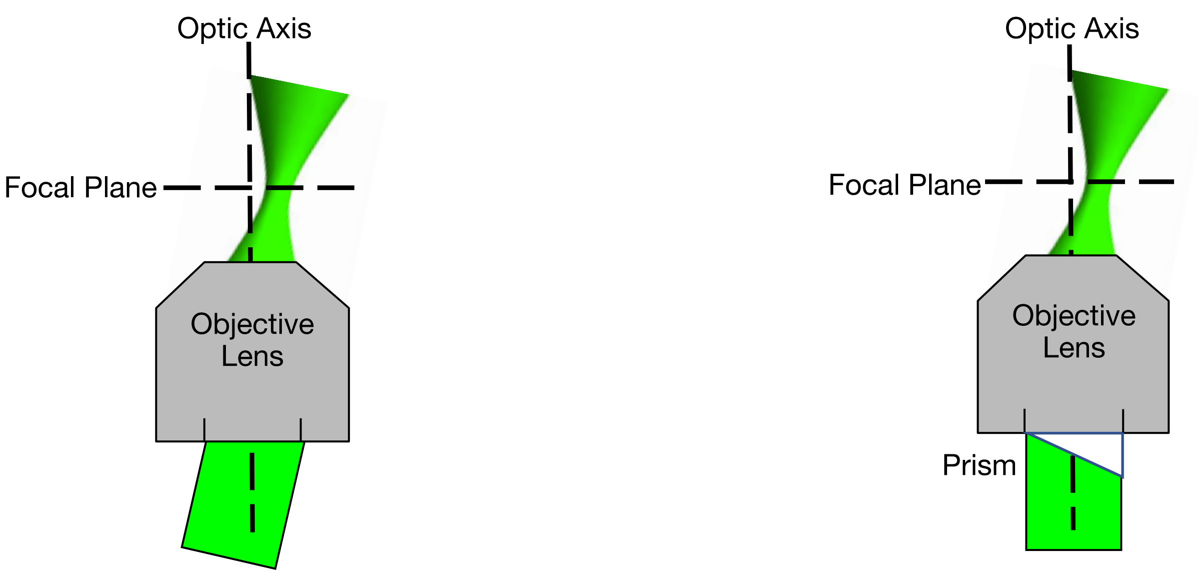

Fig. 2. At left, a beam enters, *at an angle*, the bottom of a microscope objective lens, which results in a displacement of the focal spot, away from the optic axis. At right, a prism imposes *time* delays, with the same effect.

Using an SLM, we can ***programmatically*** *determine* phase lags.

Suppose we want the light leaving an SLM to be deflected by an angle θ from the z-axis (measured towards the x-axis): the output wave vector should be $\vec{k} = k \sin\theta \hat{x} + k \cos\theta \hat{z}$. As it exits the SLM (where we label $z = z_0$), the equation describing the electric field of a plane wave then becomes, at time $t = t_0$:

$$\vec{E} = \vec{E_0} e^{i\left(\vec{k}\cdot\vec{r} - \omega t\right)} = \vec{E_0} e^{i\left(k_x x + k_y y + k_z z - \omega t\right)} = \vec{E_0} e^{i\left(k \sin\theta x + k\cos\theta z_0 - \omega t_0\right)}$$

$$= \left[\vec{E_0} e^{i\left(k\cos\theta z_0 - \omega t_0\right)}\right] e^{i\left(k\sin\theta x\right)} = \vec{E_0'} e^{i\left[k\sin\theta\right] x}$$

That is, as Fig. 1 and Fig. 2 illustrate, in order to tilt the wavefront, we simply require for the phase shift, $\delta(x)$, produced by the SLM to vary *linearly* with pixel position, x, as:

$$\delta(x) = \left[k \sin\theta\right] x = \left[\frac{2\pi}{\lambda} \sin\theta\right] x$$

Note that the pre-factor in square brackets depends on the wavelength of the light used, and the angle we desire for the output beam.

Now that you've calibrated the SLM, you know that you can impose phase profiles of this sort, …but your calibration also revealed that the device is *not* capable of producing any phase shift beyond $\sim 2\pi$, which would seem to severely limit the kinds of beam deflections possible, were it not for the fact that, for continuous wave illumination, we can write these phase profiles modulo 2π. That is, for continuous wave illumination, the effect of a 3π phase shift will be the same as a π phase shift. In other words, we can use the SLM to produce *larger-than-expected* phase shifts, so long as we use the trick of "wrapping" the phase back into the domain of $[0, 2\pi]$.

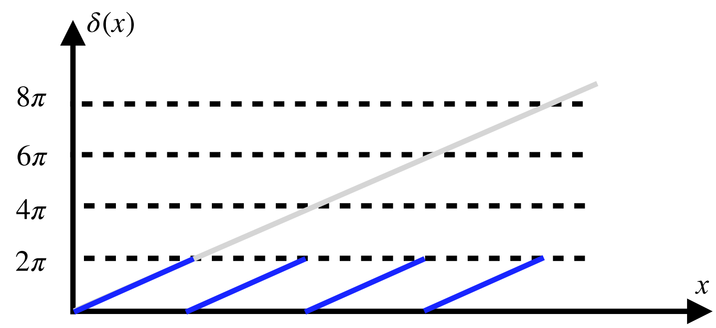

Fig. 3. Phase wrapping. As the phase shift δ increases above 2π, a multiple of 2π is subtracted (or added) to bring δ back into the domain of $[0, 2\pi]$.

In this particular example, a *linear* phase profile has been transformed into a sawtooth function.

In the image *at left* below, the phase throw generated by the SLM is a linear function *modulo* some particular grayscale level, which functions as a *virtual prism* that you create via coding:

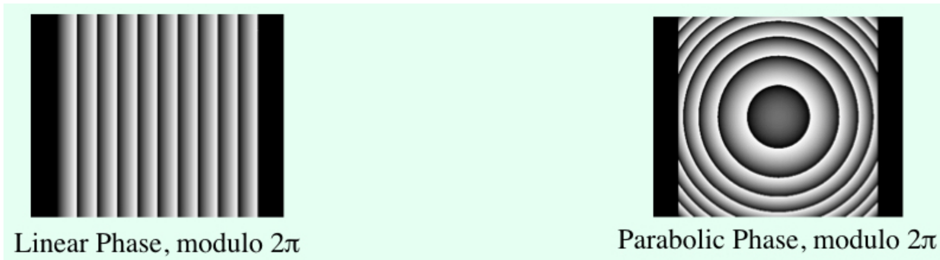

Fig. 4. At left, phase wrapping converts a prism into a "blazed diffraction grating." At right, phase wrapping converts a lens into a "Fresnel lens."

Example 2: Creating a focus

Waves spreading isotropically in 3D from a point source give rise to spherical wavefronts. (Similarly, by time reversal symmetry, if we reverse the flow of those wavefronts, they would converge to a focus.) You can *program* the SLM to add curvature to the outgoing wavefronts. Your text argues that the equation for the electric field associated with an outgoing spherical wave, at a distance r from its origin, is:

$$\vec{E} = \frac{\vec{A}}{r} e^{i(kr-\omega t)}$$

The factor of $1/r$ in the prefactor is required by conservation of energy. (Isotropic spreading in 3D means that the energy goes as $1/r^2$, so the field must goes as $1/r$.) For now though, we want to keep things simple and use the SLM as a Phase-*Only* Modulator. So, let's just assume that r is large enough, and the SLM's active area is small enough, that the prefactor will not vary much across the SLM. In that case, we can write the prefactor as $\vec{A}/z_0$. In the limit of interest $z_0 \gg \lambda$, even though the amplitude does not vary significantly across the SLM surface, the phase *does*, so we *cannot* simply treat the factor of r in the phase as if it were constant:

$$\vec{E} = \frac{\vec{A}}{z_0} e^{i\left[k\sqrt{x^2+y^2+z_0^2}-\omega t\right]} = \left[\frac{\vec{A}}{z_0}e^{-i\omega t}\right] e^{ikz_0\left\{1+\left[\left(\frac{x}{z_0}\right)^2+\left(\frac{y}{z_0}\right)^2\right]\right\}^{1/2}}$$

When the SLM is far from the origin of the sphere, $z_0 \gg x, y$ and so we can use the binomial expansion to simplify our result at, say, $t = t_0$, to:

$$\vec{E} = \left[\frac{\vec{A}}{z_0}e^{-i\omega t_0}\right] e^{ikz_0\left\{1+\frac{1}{2}\left[\left(\frac{x}{z_0}\right)^2+\left(\frac{y}{z_0}\right)^2\right]\right\}} = \left[\frac{\vec{A}}{z_0}e^{i(kz_0-\omega t_0)}\right] e^{i\left[\frac{k}{2z_0}\left(x^2+y^2\right)\right]} = \vec{E}_0 e^{i\left[\frac{k}{2z_0}\left(x^2+y^2\right)\right]}$$

That is, the phase factor that we are concerned with is *parabolic* in x and y:

$$\delta(x,y) = \frac{k}{2z_0}\left(x^2+y^2\right)$$

Thus, a *virtual lens* can be generated by properly translating this phase profile into grayscale image, such as the one shown on the right side of Fig. 4 above. Or, to be more precise, we can convert from a diverging lens to a converging lens simply by changing the sign of the curvature.

To review, when we replaced the factor of $1/r$ in the amplitude with z_0, we were assuming that r is "large enough," and the SLM's active area is "small enough;" we were essentially making a *small-angle* approximation, where none of the rays of relevance deviate significantly in angle from the optic axis. We then made a related approximation, $z_0 \gg x, y$, allowing us to use the binomial expansion on the phase factor. Together, these define the "**Paraxial Optics**" limit, which yields *many* simplifying results and tools that we will encounter in this course. At the end of the course, though, we will emphasize situations, such as high-power microscope objective lens shown in Fig. 1, where the results of Paraxial Optics are *not* applicable, so it will pay to keep in mind that approximations have been made here, *e.g.*, if you wish to make optical traps.

Example 3: Creating a cylindrical lens

Happily, a cylindrical wave will have a phase relationship just like the one above but, say, with a dependence only on x or y, rather than on both.

Because SLM devices provide for simple programmatic control of phase (and/or amplitude or polarization) within many local regions across the field of a beam of light, they clearly allow for *direct*, *systematic* exploration of all of the mathematical models contained in the text (including those described above, and much, much more). In this way, the mathematical models of Optics become hands-on tools for exploring the physical world and, so long as *they are programming* these models into the SLM, it is our expectation that students will be better able to come to terms with these models and, ultimately, will be more likely to bring the power of these mathematical tools to bear within the context of their own projects.

The Experiment: Basic examples of Wavefront Modification

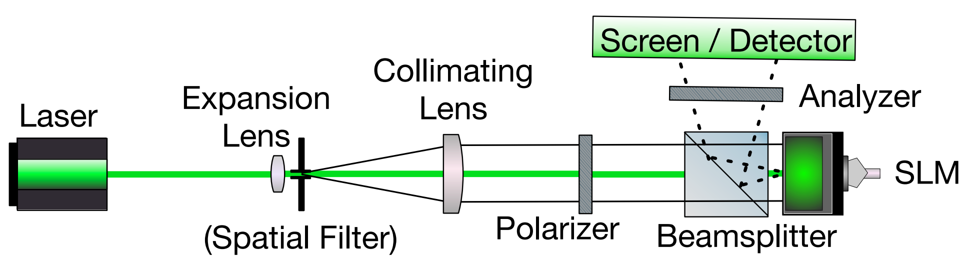

Fig. 5. Your setup from last week can now be utilized for creating angular beam deflections and/or spherical and/or cylindrical waves

Using the apparatus shown in Fig. 5, the Polarizer and Analyzer should have their transmission axes oriented so as to optimize *Phase-Only* Modulation. Refer back to your lab notebook and/or reading about Digital Optics ("Case B").

Your notebook should include **images** of the applied phase profiles and **descriptions** of the results. Create and apply a variety of simple phase patterns on the SLM:

A) Predict the outcome, and then apply a linear phase profile. Celebrate! Apply profiles to deflect a plane wave to the left, and then to the right.
B) Explore the upper limits on the angle of deflection. Whatever you see, say what you saw, in your lab notebook, then go on to describe your thoughts regarding these limits.
C) Review the prefactors contained in the models described above, as well as your results from the Week 2 lab on "Diffraction Noise," regarding pixelation. Attempt to *model* the limits you have encountered in this lab.
D) Predict the outcome, then apply a phase profile intended to create converging spherical waves. Celebrate. Then create diverging spherical waves. Celebrate.
E) Explore the consequences of applying extreme curvatures. Whatever you see, say what you saw, in your notebook, then go on to describe your thoughts regarding these effects.
F) Predict the outcome, and then create a converging cylindrical wave. Celebrate.
G) Phase profiles can be also *added* modulo 2π. In this way, combine a phase profile used to create a converging spherical wave with a phase profile used to laterally deflect a beam. Describe your composite phase profile. Does the composite phase profile look the way you think it should? Continuing to combine simple phase profiles in this way allows you create *multiple* focal spots, distributed in 3D. These non-iterative algorithms constitute your first approach simple Computer-Generated Holograms (CGH), …but you'll explore other approaches in a few weeks, and then again later in the course.
H) In class, we can explore the app *iHologram*, and comment in your lab notebook upon your observations. Note that when three linear phase profiles ("blazes") are superimposed, there will generally be "singular points," where *all* phases meet. Phases range from 0° to 360°, so in terms of phasors, when all phases are present 0° and 180° will cancel out, 90° and 270° will cancel, and so on: by symmetry, singular points must yield dark output spots.

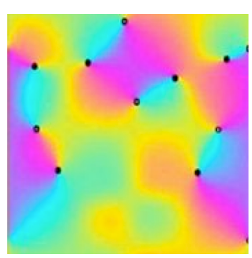

Fig. 5. **Phase Singularities** are ubiquitous. Here, color is used to represent the local phase. Dark spots have been added to this profile, to accentuate points where all phases meet. [Image: Miles Padgett, Univ. of Glasgow]

Opportunities for Initiative: (your own ideas may be better!)

— You could create a grayscale image that encodes some other phase profile that you come up with (an azimuthal gradient? a radial gradient? a cubic phase profile? low-order "Zernike polynomials"?). Come up with a procedure to use, to analyze the light that is output, and be sure to predict the results before applying your profile; only then will you be able to celebrate the outcome! (If your prediction is proven correct, you have the right to celebrate; if nature surprises you, this, too, is worth celebrating! Nature is marvelously full of wonder!)

— You could even create an image that divides the SLM into zones, applying an array of phase profiles (*e.g.*, you could create a lenslet array, or whatever array of phase profiles you wish). This is sometimes referred to as *multiplexing*. One application: while LCoS SLMs update slowly, in our labs you can use an Acousto-Optic Deflector (AOD) to step a beam across a multiplexed SLM, at rates up to 300000 Hz. — Acousto-Optic devices are available in the lab (or can be obtained for free by scavenging parts from some commercial laser printers, once they are discarded by the school.).

> CAUTION: When reducing the input beam area, to fit into smaller zones on the SLM, always be mindful that you must remain below the **Damage Threshold** of the SLM.