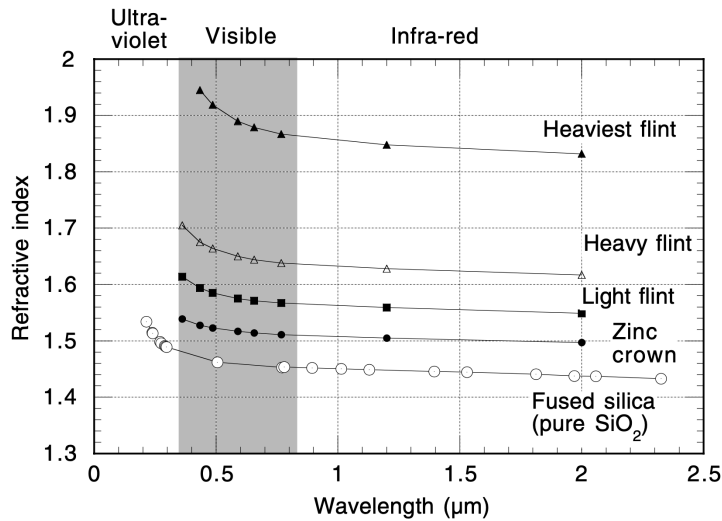


## Gabe's simple model for the wavelength dependence of an SLM:

The *maximum* phase shift that can be produced by an SLM depends upon the thickness of the liquid crystal,  $t$ , and upon the wavelength of light used:

$$\phi_{max} = 2\pi \frac{t}{\lambda/n}$$

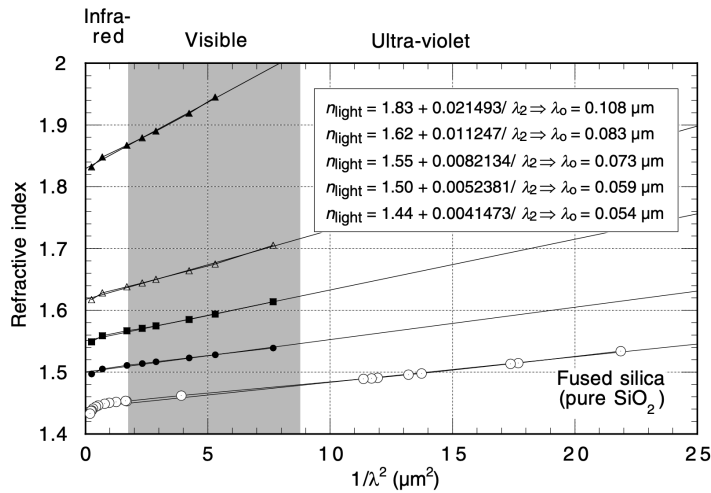
However, the index of refraction of the liquid crystal material also has a wavelength dependence! The (open-source) textbook that I use for teaching Materials Physics, [Understanding the Properties of Matter](#), shows this for a variety of optical materials:



In fact, that textbook derives a simple approximate model by treating the sloshing of electrons in a material by using “Hooke’s Law,” yielding a simple prediction for the index of refraction:

$$n = n_{\lambda=\infty} \left[ 1 + \left( \frac{\lambda_0}{\lambda} \right)^2 \right]$$

This simple *quadratic* form is what you get for any resonance in a linear system (e.g., a driven mechanical oscillator), when you are approaching resonance “from one side,” but are still “**far from**” resonance. It is an equation with two *adjustable parameters*: the prefactor,  $n_{\lambda=\infty}$ , is clearly the value yielded for the index of refraction when you plug in an “infinite wavelength” (hence the name). If you prefer to call it “Rufus Scrimgeour,” and to call the other fitting parameter ( $\lambda_0$ , which happens to have units of wavelength) “Barty Crouch, Sr.,” it would make no difference: they are both just “Fudge Factors” that differ from material to material. The prefactor,  $n_{\lambda=\infty}$ , is larger for materials where electrons are less tightly bound. The fitting parameter with units of wavelength,  $\lambda_0$ , varies as you consider materials where an electron resonance is closer or further away. (For transparent materials, electron resonances are *typically in the ultraviolet*.)



In the Materials Physics textbook, the data shown in the previous figure is re-plotted in a manner that helps to support the simple prediction. Plugging this generic form in, we can re-write the maximum phase shift given by the SLM as:

$$\phi_{max} = \left(2\pi \frac{t}{\lambda}\right) n_{\lambda=\infty} \left[1 + \left(\frac{\lambda_0}{\lambda}\right)^2\right].$$

For simplicity, we introduce a constant  $T$ , which depends upon the liquid crystal thickness (which in turn *determines* the optimal operating wavelength for that SLM):

$$T \equiv (2t) n_{\lambda=\infty}$$

This change of variables serves to *replace* the prefactor,  $n_{\lambda=\infty}$ , from our equation for the index of refraction, with something that you might wish to consider an “**effective optical path length**,”  $T$ . This, too, is **just an adjustable fitting parameter**, but it does yield our *preferred* form for the “phase throw” of the SLM:

$$\phi_{max} \text{ (in } \pi \text{ radians)} = \frac{T}{\lambda} \left[1 + \left(\frac{\lambda_0}{\lambda}\right)^2\right].$$

For an input grayscale level of 255, a “loaner” SLM that I borrowed from Hamamatsu gave the following (experimentally determined) phase throws as a function of laser wavelength:

$\lambda$ (nm)	$\Phi$ (in $\pi$ radians)
400	4.69
500	3.09
600	2.28
633	2.13
700	1.88

This data set yields a **best-fit value for  $\lambda_0$  of 373.78 nm**, which I use to describe all SLMs using that particular liquid crystal solution.

My friend Kishan Dholakia's "780nm" SLM (from the same manufacturer Hamamatsu, S/N XL\_0043 with front end S/N PS00646), yields a (measured) maximum phase throw of  $2.16\pi$ . That is, for its *design wavelength* of  $\lambda = 780\text{nm}$ ,

$$2.16\pi = \frac{T}{\lambda} \left[ 1 + \left( \frac{373.78\text{nm}}{\lambda} \right)^2 \right]$$

which yields a **best-fit value of  $T = 1370.2\pi$  radians**.

Armed with these best-fit values for the two adjustable parameters in our model, for a liquid crystal material, our simple model can be used to *predict* the maximum phase throw that will result when we use this SLM with, say, 633nm light:

$$\phi_{max}^{633nm} = \frac{T}{\lambda} \left[ 1 + \left( \frac{\lambda_0}{\lambda} \right)^2 \right] = 2.9\pi \text{ radians.}$$

Therefore, at this new wavelength, our model predicts that the ***max grayscale level*** corresponding to  $2\pi$  would now be:

$$255 \left( \frac{2\pi}{2.9\pi} \right) = 174$$

That is, when used at a laser with a wavelength of 633nm, ***instead of*** creating images that have **grayscale levels going from 0-255, those grayscale levels should go from 0-174**.

Note also that my model predicts that this SLM might also reasonably be used for *longer* wavelengths, all the way up to 1460nm, where the maximum phase throw is  $\pi$  (and you would then operate it to produce either amplitude modulations or ***binary*** phase modulations, which don't have the same efficiencies as what we're used to, but can still be *plenty* useful).