The RC Circuit

Circuits containing both resistors and capacitors have many useful applications. Often RC circuits are used to control timing. Some examples include windshield wipers, strobe lights, and flashbulbs in a camera, some pacemakers. One could also use the RC circuit as a simplified model of the transmission of nerve impulses.

Theory

Figure 1 shows a simple circuit consisting of a capacitor, *C*, a resistor, *R*, a "double-throw switch," S and an external power supply. Initially, the capacitor is uncharged and the switch is in the middle position.

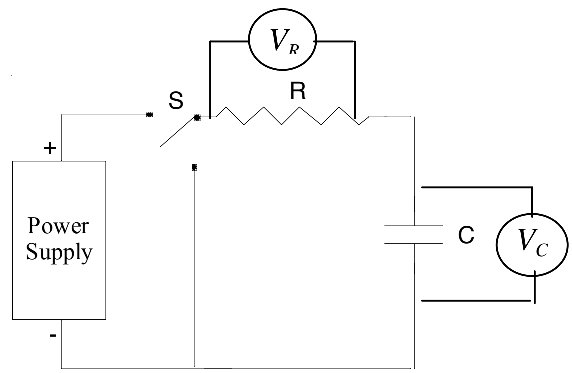

Figure 1

Suppose that the switch is pushed "upward" in Figure 1, connecting the battery to a simple "closed circuit." In this case, the resistor and capacitor are connected in *series* with the power supply. The resistor limits the rate at which charges reach the capacitor, an effect we will study in this lab. When the switch is thrown at time t =0, the capacitor is initially uncharged, so we do not have an equilibrium situation: there is now a potential difference between the power supply and the capacitor. Consequently, charges at the power supply experience a force and flow from the power supply, through the resistor, and accumulate on the capacitor. As the net charge on the capacitor increases, the potential difference across the capacitor increases and consequently fewer charges are able to flow to the capacitor. Note that as the potential across the capacitor increases, the potential across the resistor decreases. Eventually, there are so many charged particles on the capacitor that it is at the same potential as the power supply and no additional charges flow (and the potential across the resistor would become negligible). This is the equilibrium situation.

In nature, we often find circumstances where the *rate of change* in some quantity, in this case charge, is proportional to that quantity's instantaneous value. In such cases, the quantity is always described by an exponential function in time. Using calculus, one can find the number of charged particles on the capacitor. The result of this calculation is

$$Q_C(t) = CV_{PS}\left(1 - e^{-t/RC}\right) \qquad (1)$$

where $Q_C(t)$ is the charge on the capacitor, at time t, V_{PS} is the voltage across the power supply, R is the resistance of the resistor and C is the capacitance of the capacitor. The product RC turns out to have units of *time*, and is referred to as the "time constant" of the circuit.

It follows that the potential difference, or voltage, across the capacitor is:

$$V_C(t) = V_{PS}\left(1 - e^{-t/RC}\right) \tag{2}$$

where $V_C(t)$ is the voltage across the capacitor at some time, t.

Likewise we could measure the voltage across the resistor and we would find

$$V_R(t) = V_{PS}e^{-t/RC} \tag{3}$$

Qualitatively explain why you might expect equation (3) to be valid when charging the capacitor.

Once the capacitor is fully charged (or about ~ 99% charged, after a duration equal to five time constants has passed), we can remove the power supply by putting the switch in the "down" position in Figure 1. We now have removed the battery from the circuit, leaving the capacitor wired in parallel with the resistor. At any point in time, the potential difference across the capacitor is the same as the potential difference across the resistor, $V_R(t)$. There is, when we first move the switch to this position, a potential difference across the capacitor because it is fully charged, and so charges will flow from one side of the capacitor, through the resistor, to the other side of the capacitor. From Ohm's Law, we know that the current, $I_R(t)$, flowing through the resistor, R, at some time, t, is

$$I_R(t) = \frac{V_R(t)}{R} \tag{4}$$

Physically, current is the "flow" of charge. In this case, it is the *rate* at which charge is leaving the capacitor. Consequently, the potential difference across the capacitor decreases in time. As the potential difference decreases, so does the current, as one would expect from Ohm's Law. We again are faced with a situation where the rate of change of the charge stored on the capacitor is proportional to the instantaneous charge. With a little work, one can show that the potential difference across the capacitor, as the capacitor discharges, is described by the following expression:

$$V_C(t) = V_{PS}e^{-t/RC} \tag{5}$$

It is interesting that the rate of discharge depends only upon the product of R and C, which, again, is called the time constant, $\tau = RC$. At time $t = \tau$, the voltage is precisely e^{-1} of its original value.

$$V(t = \tau) = V_{PS}\, e^{-1} = 0.368\, V_{PS} \tag{6}$$

The time constant is a characteristic timescale of any RC circuit. Note, however, that Equation 5 implies that the voltage *never* becomes zero but only approaches it asymptotically; of course, for practical purposes the voltage becomes negligibly small after a period equal to a few time constants. **Five time constants** is generally regarded as being the time required to charge or discharge a capacitor (99% charged or discharged).

Procedure

Part 1: Charging & Discharging

With *Data Studio* you can knock out both the classic cases of charging and discharging in "one fell swoop". We begin by examining the potential difference across a capacitor and a resistor as the capacitor charges using Pasco's Science Workshop and the circuit shown in Figure 1. At your table, there are a collection of resistors and capacitors. Use at least two resistor/capacitor combinations with significantly different time constants. After you have wired the circuit shown in Figure 1 have it checked by the instructor or TA.

Record the *RC* combinations that you use. **Include an estimate of the expected duration for about six different time constants.**

Depending on the chosen values of *R* and *C* that you have selected, you will need to take data for varying lengths of time. For instance, if you chose $R = 10\ k\Omega$ and $C = 100\ \mu F$, the RC constant would be 1 second and you need to take data for at least 5 seconds. Similarly, the duration of the experiment could be on the order of milliseconds. Because of this potential disparity in experiment length, you will need to tell the computer how often you want to record data; configured via setting the "*Sampling Rate*." -- as we did last semester. (Note that the maximum sampling rate sets a minimum measureable limit on the time constants you can use.)

Ideally, you would like to have at least 50 or 60 data points during the discharge of the capacitor. For instance, if you chose $R = 10\ k\Omega$ and $C = 100\ \mu F$, then a sampling rate of 10 *Hz* would be appropriate. This would record 10 data points every second, leaving you with 50 data points during the discharge of the capacitor.

It may also be useful to tell the computer when to start and stop taking data. This is particularly useful if you have chosen a very short *RC* time and you do not want to sort through a large amount of uninteresting, and extraneous, data. To do this, well, … figure it out!

We are now ready to take some data. Turn on the power supply and adjust the power supply voltage to **just barely *under* 10 volts**. The capacitor will begin to charge when the switch is closed such that the resistor and capacitor are in series (the "upper" position in Figure 1).

You may need to observe a given capacitor's charging a couple of times, before you obtain a "nice-looking" result. Thus, it is suggested that you start with the combination having a fairly short *RC* time constant. You should continue taking data until the capacitor is fully "charged."

Once you have taken the data, plot V/V_o versus t. You have a model, Equation 2, which predicts what your data should look like. For comparison, you may want to simulate Equation 2 on the same graph using the nominal values of *R* and *C*, which are marked on these components and also a simulation based on the static measurements of the "isolated" components. These graphs should be printed, cut and then taped into your lab notebook.

<u>Does your data agree with the model given by our theory? Discuss any discrepancies and what might account for them. How does the experimental time constant compare with your expectation?</u>

Questions

1) Show that the time constant has the dimensions of seconds when *R* is expressed in *ohms* and *C* in *farads*.

2) A charged 2 μF capacitor is connected in parallel with a 5000 $k\Omega$ resistor. How long after the connection is made will the capacitor voltage fall to:
   (a) 50% of its initial value
   (b) 30% of its initial value
   (c) 10% of its initial value
   (d) 5% of its initial value?

3) Is charge conserved? Support your answer with the data you have taken.

Initiative:

Possible ideas:

1. Using what you have done in this lab, verify the behavior of capacitors in series and parallel that is provided in your text (this is the "weakest" case!).

2. Discuss the effects that the 1 $M\Omega$ internal resistance of the voltage inputs of the Science Workshop interface has on the behavior of the *RC* circuit.

3. How much charge flowed through the resistor during the charging/discharging of a capacitor? Graphically estimate this value and compare it with what you would expect, numerically.

4. Use the function generator (ask the TA or instructor) and build an "integrator" and/or "differentiator."

Conclusion:

Write one!