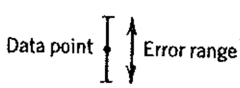
GRAPHICAL ANALYSIS

(Excerpted from "The Art of Experimental Physics," by Preson & Dietz)

A purpose of many experiments is to find the relationship between measured variables. A good way to accomplish this task is to plot a graph of the data and then analyze the graph. These guidelines should be followed in plotting your data:

1. Use a sharp pencil or pen. A broad-tipped pencil or pen will introduce unnecessary inaccuracies.
2. Draw your graph on a full page of graph paper. A compressed graph will reduce the accuracy of your graphical analysis.
3. Give the graph a concise title.
4. The dependent variable should be plotted along the vertical (y) axis and the independent variable should be plotted along the horizontal (x) axis.
5. Label axes and include units.
6. Select a scale for each axis and start each axis at zero, if possible.
7. Use error bars to indicate errors in measurements, for example,



8. Draw a smooth curve through the data points. If the errors are random, then about one-third of the points will not lie within their error range of the best curve.

The microcomputer is a powerful tool for data analysis. Commercial software is available that handles data and instructs the microcomputer to carry out graphical analysis. See your instructor about the availability of this software for your laboratory.

As an example consider the study of the speed of an object (dependent variable) as a function of time (independent variable). The data are as follows:

Speed (m/s)	Time (s)
0.45 ± 0.06	1
0.81 ± 0.06	2
0.91 ± 0.06	3
1.01 ± 0.06	4
1.36 ± 0.06	5
1.56 ± 0.06	6
1.65 ± 0.06	7
1.85 ± 0.06	8
2.17 ± 0.06	9

Using the above guidelines, the data are graphed in Figure I.7.

The graphed data show that the speed v is a linear function of the time t. The general equation for a straight line is

$$y = mx + b \tag{22}$$

where m is the slope of the line and b, the vertical intercept, is the value of y when $x = 0$. Let $v = y$,

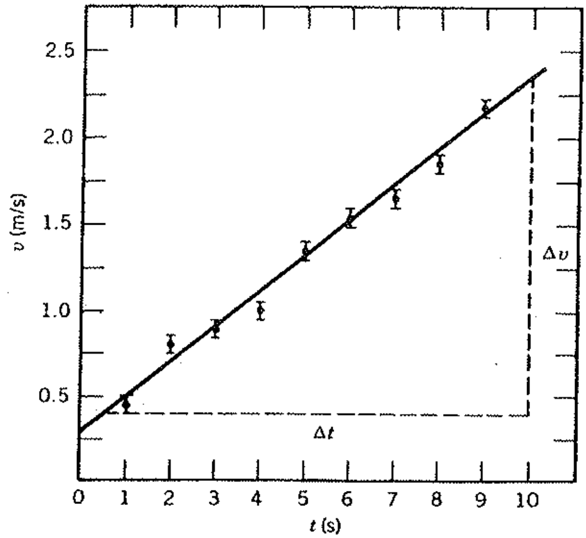


FIGURE I.7 Speed versus time. The graphed data, v versus t, show a linear relation.

$x = t$, $a = m$, and $v_0 = b$; then,

$$v = at + v_0 \qquad \text{(m/s)} \qquad (23)$$

This is the form of the equation for the line drawn through the data, where v_0 is the value of the velocity at $t = 0$ and a is the slope of the line that is the acceleration of the object. From the graph we see that $v_0 = 0.32$ m/s. To determine the slope select two points on the line, but not data points, which are well separated, then

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{2.35 - 0.40 \text{ (m/s)}}{10.0 - 0.5 \text{ (s)}}$$

$$= \frac{1.95 \text{ (m/s)}}{9.5 \text{ (s)}} = 0.20 \text{ m/s}^2 \qquad (24)$$

The equation for the line is

$$v = 0.20t + 0.32 \qquad \text{(m/s)} \qquad (25)$$

The data plotted in Figure I.7 are analyzed in the section on "Curve Fitting," page 23, as an example of linear regression.

As a second example, let us consider the study of the distance traveled by an object as a function of time. The data are as follows:

Distance (m)	Time (s)
0.20 ± 0.05	1
0.43 ± 0.05	2
0.81 ± 0.05	3
1.57 ± 0.10	4
2.43 ± 0.10	5
3.81 ± 0.10	6
4.80 ± 0.20	7
6.39 ± 0.20	8

The data are graphed, using the above guidelines, in Figure I.8.

In this instance a straight line through the data points would not be acceptable. An inspection of the graph suggests that d is proportional to t^n, where $n > 1$; for example, d may be a quadratic function of time and, hence, $n = 2$.

Suppose that we know the theoretical relation between d and t is

$$d = \tfrac{1}{2}at^2 \qquad \text{(m)} \qquad (26)$$

where a is the object's acceleration. Often it is useful to know if the data agree with the theory. If the data follow the above theoretical relation, then a graph of d versus t^2 should result in a straight line.

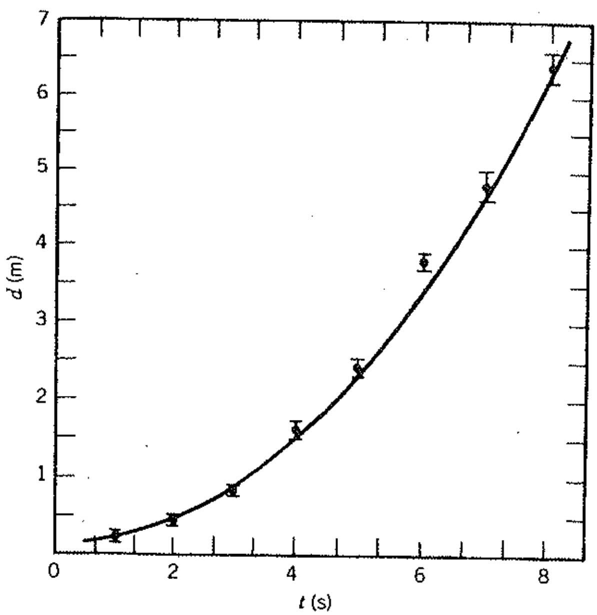


FIGURE I.8 Distance versus time. The graphed data, d versus t, show a nonlinear relation.

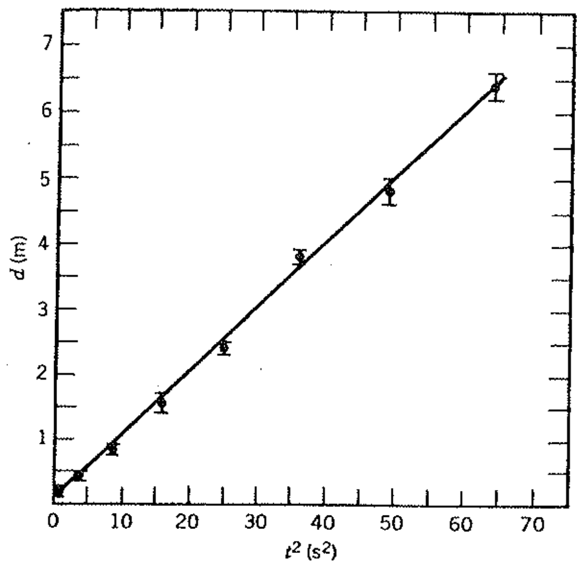


FIGURE I.9 Plotting d versus t^2 yields a linear relation.

The graph in Figure I.9 indicates that d is a linear function of t^2 and, hence, that the data agree with the theoretical relation. The equation for the straight line is

$$d = mt^2 + d_0 \qquad \text{(m)} \qquad (27)$$

where m is the slope and d_0 is the vertical intercept.

PLOTTING DATA ON SEMILOG PAPER

Often the relationship between the measured variables is not linear. For example, consider the intensity of light I transmitted through a sample of thickness x, shown in Figure I.10, where I_0 is the incident intensity of the light.

Lambert's law states the theoretical relationship between the dependent variable I and the independent variable x:

$$I = I_0 e^{-\mu x} \qquad (\text{W/cm}^2) \qquad (28)$$

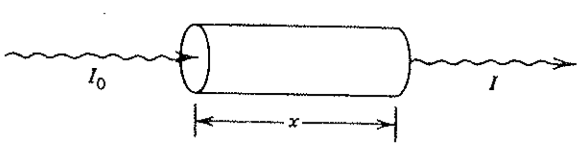


FIGURE I.10 I_0 is the incident light intensity, x is the sample thickness, and I is the transmitted intensity.

where μ is the absorption coefficient, a constant that depends on the wavelength of light and the absorbing properties of the sample. Suppose I is measured as a function of x, and the data are plotted as is shown in Figure I.11.

From the smooth curve it would be difficult to determine the relationship between I and x, that is, it would be difficult to conclude the data obey Lambert's law.

A good way to determine the experimental relationship between I and x is to use semilog paper. Semilog paper has a logarithmic y axis (it automatically takes logarithms of data plotted) and a regularly spaced x axis. The data are plotted on semilog paper in Figure I.12. Note that there is never a zero on the logarithmic axis, and that when reading values off of a logarithmic axis you read the logarithm of the value and not the value, for example, log 9 and not 9.

Homework

Q: Is this true ??

Discuss

The smooth curve drawn through the data is a straight line with a negative slope and the intensity at the point on the vertical axis intercepted by the curve is I_0. Lambert's law does agree with this result as can be seen by taking the logarithm of Lambert's law:

$$\begin{aligned} \log I &= \log(I_0 e^{-\mu x}) \\ &= \log e^{-\mu x} + \log I_0 \\ &= -\mu x \log e + \log I_0 \\ &= -0.434\mu x + \log I_0 \qquad \text{(unitless)} \quad (29) \end{aligned}$$

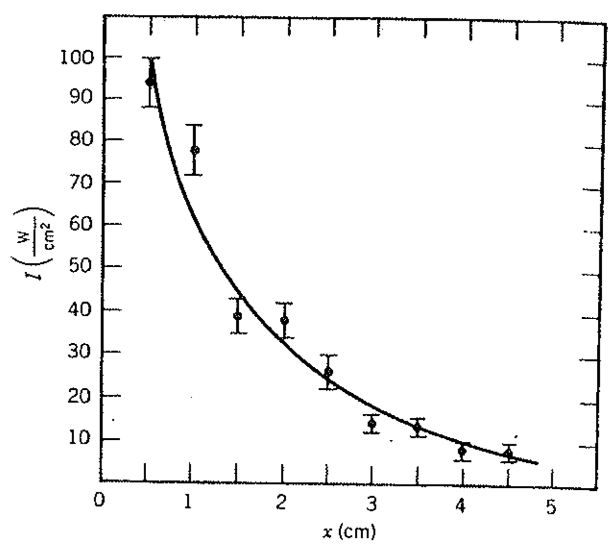


FIGURE I.11 Light intensity versus sample thickness, showing a nonlinear relation. From the graph it is not clear if the data obey Lambert's law or not.

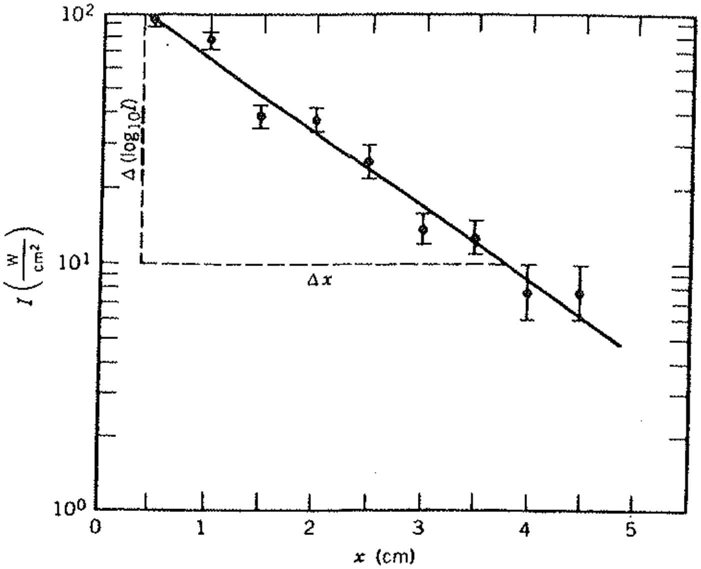


FIGURE I.12 Light intensity versus sample thickness. The linear relation obtained on semilog paper shows that the data obey Lambert's law.

Again, the general equation of a straight line is of the form:

$$y = mx + b \tag{30}$$

Now let $y = \log I$, $m = -0.434\mu$, and $b = \log I_0$. Then, if $\log I$ is plotted vertically and x is plotted horizontally, the curve will be a straight line with slope -0.434μ and vertical intercept $\log I_0$. Using semilog paper, I is plotted on the logarithmic axis; the vertical intercept on this axis is I_0. Note that the slope of the line drawn through the data points may be used to calculate μ:

$$\text{slope} = \frac{\Delta(\log I)}{\Delta x} = \frac{\log 10 - \log 100}{(3.80 - 0.40)\text{ cm}} = -0.294\text{ cm}^{-1} \tag{31}$$

From Lambert's law the theoretical slope is

$$\text{slope} = -0.434\mu$$

By equating theoretical and experimental slopes, we find that

$$-0.434\mu = -0.294\text{ cm}^{-1}$$

and

$$\mu = +0.678\text{ cm}^{-1}$$

EXERCISE 2

Homework

Suppose the functional relation between the dependent variable y and the independent variable x is given by

$$y = a\,e^{-x} + b$$

where a and b are nonzero constants. Explain why a graph of y versus x on semilog paper would not give a straight line.

Write your answer:

PLOTTING DATA ON LOG–LOG PAPER

Log–log paper is used to obtain a straight line plot when y and x satisfy a power-law relation:

$$y = cx^n \tag{32}$$

This is the definition of a "Power Law"

where c and n are constants. For example, the semimajor axis R of the orbit of a planet is related to its period (time for one revolution around the sun) T:

$$R^3 = KT^2 \quad \text{or} \quad R = K^{1/3}T^{2/3} \tag{33}$$

where K is a constant. R is nonlinearly related to T.

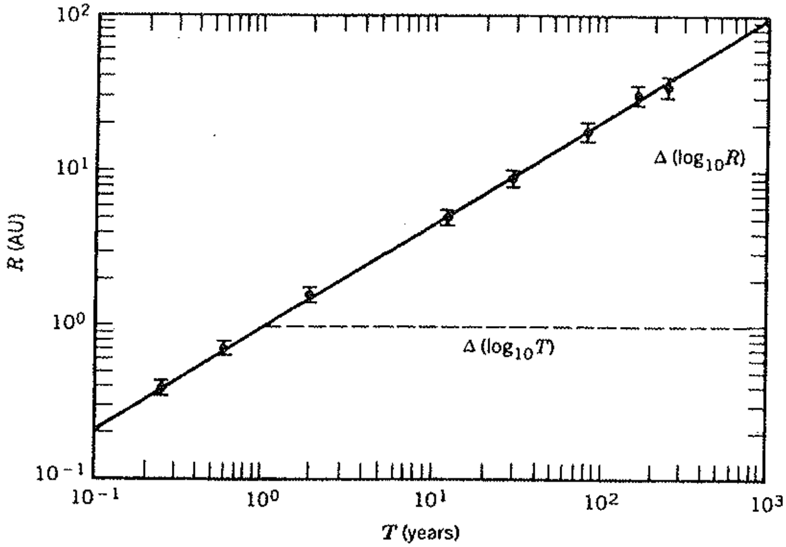


FIGURE I.13 Planets: Semimajor axis versus period. The linear relation on log–log paper indicates R and T obey a power law of the form of equation 32.

Homework

A straight-line plot is obtained in the following way. Take logarithms

$$
\begin{aligned}
\log R &= \log(K^{1/3}T^{2/3}) \\
&= \log T^{2/3} + \log K^{1/3} \\
&= 2/3 \log T + \log K^{1/3} \qquad (34)
\end{aligned}
$$

Let $y = \log R$, $x = \log T$, $m = \frac{2}{3}$, and $b = \log K^{1/3}$. Then a plot of $\log R$ versus $\log T$ would be a straight line. Log–log graph paper automatically takes the logarithm of the plotted data. A log–log graph is shown in Figure I.13.

The units used are years and astronomical units (AU), where 1 AU is the semimajor axis of earth's orbit. (The errors shown in the graph are fictitious.) The slope of the log–log plot is

$$
\begin{aligned}
\text{slope} &= \frac{\Delta(\log R)}{\Delta(\log T)} = \frac{\log 10^2 - \log 10^0}{\log 10^3 - \log 10^0} \\
&= \frac{2-0}{3-0} = \frac{2}{3} \qquad (35)
\end{aligned}
$$

Note that the slope of the log–log plot is the exponent of the power law relation. For example, the power law relation $y = cx^n$ plotted on log–log paper has a slope equal to n. Hence, a log–log plot is a good way to determine the exponent in a power law relation.

Another way to obtain a straight-line plot is to plot y versus x^n or R versus $T^{2/3}$ on regular graph paper (see Figure I.14).

You should Discuss:

A problem with plotting R versus $T^{2/3}$ is that values of R less than about 1 AU cannot be plotted with much accuracy.

In units of years and astronomical units the constant K is one, and an inspection of the curve in the figure shows a slope of approximately one.

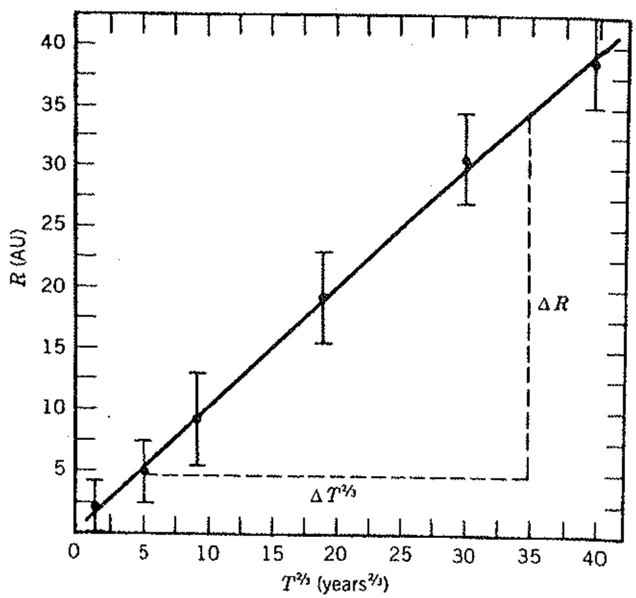


FIGURE I.14 Planets: R versus $T^{2/3}$, showing a linear relation. This graph requires knowing the exponent in the power-law relation.