Physical Optics (considering *phase shifts*)

**Background**
An interesting optical effect stands out when light encounters a small aperture or barrier, such as a pinhole or a fine slit or single strand of hair. This effect is known as diffraction and is a consequence of the fact that light propagates like a wave (and *not* in the way suggested by the simple ray diagrams used in the geometric optics approximation): in particular, we will see that *phase shifts* (whether due to differences in path length or in the media traversed) can have enormous consequences on the transmitted beam shape.

**Procedure**
The experimental setup is shown in Figure 1. A monochromatic Helium-Neon laser, having a known wavelength of $\lambda = 632.8 \text{ nm}$, is positioned on the short bench. The laser should be positioned such that it points to the center of the screen. Objects causing diffraction will be positioned just a few centimeters from the laser. To get the greatest resolution, the (measured) **distance between the object and the observing screen, $L$, should be as large as possible.** (For very small apertures, the pattern gets too dim to observe when it spreads out over large distances.)

![Figure 1](image)

**Part 1: Circular** Apertures
Each station has been supplied with *two* small circular apertures. At most stations, these are *both* located on Slide 9165-D. Finding these tiny apertures is akin to finding a needle in a haystack; it is the hardest part of this lab, so be sure to celebrate when anyone successfully aligns one with the laser beam. (Masking tape has been used to cover, temporarily, parts of the slide that are not in use for this part of your lab work; please leave that in place.) In any case, your first step is to introduce the **larger** of these two small circular apertures into the beam path, and to **record** your estimate of the **size** of the resulting spot on your imaging screen.

Aperture diameter = _____________
$L = _____________$
Resulting spot diameter = ___________
Attempt to repeat this for your smaller circular aperture:

\[
\text{Aperture diameter } = \_\_\_\_\_\_\_\_\\nL = \_\_\_\_\_\_\_\_\\n\text{Resulting spot diameter } = \_\_\_\_\_\_\_\_\]

*Qualitatively*, how does the spot size on your imaging screen scale with the diameter of the circular aperture?

This effect (diffraction) is of practical importance, as it places a limit on how well an optical device, such as a microscope or a telescope can work. Surprisingly, when light passes through a lens, a smaller **focal spot** results when a larger beam is incident upon the lens. So, to achieve the smallest spot size for high-resolution microscopy, we need a large-diameter lens, in order to accept a large-diameter beam.
High-resolution microscope objective lenses are sold according to their “Numerical Aperture” size, but the effective resolution depends not just upon the lens itself, but upon how it is used. – In the figure above, the beam entering the objective lens is not as large as it could be, given the extent of the “entrance pupil” of the lens; in this case, we say that the lens is “underfilled” and the result is that the focal spot is not as small as it could be.

For those interested, an interactive microscopy simulation is available online at:  

Surprisingly, the same effect (diffraction) that limits our ability to directly image small things can also put to use in ways that allow us to (indirectly) probe the structure of very small things, such as the arrangement of atoms, or molecules, in a crystal and complex molecules, such as DNA or a protein.

Part 2: Linear Apertures

In the next part of this lab, we examine this effect introducing pinhole-scale linear slits into our optical path (mostly because they are easier to “find” / align to the laser).

We will use the observed diffraction patterns that result from slits of known sizes to extract an experimental measurement of the wavelength of your laser, then turn the effect around and use your measured wavelength and the diffraction pattern that results from placing a single strand of hair in the beam to measure the width of that strand of hair very
precisely. Finally, we will move on to examine the diffraction pattern of some two-dimensional structures.

If the aperture is a single linear slit, then the diffraction will produce a linear pattern with multiple maxima separated by nodes, as seen in Figure 3. The largest maximum is known as the central fringe.

Why?

Again, everything we are exploring this week is a consequence of the fact that light does not propagate in the way suggested by the simple ray diagrams used in the geometric optics approximation.

Instead, light propagates in a wave-like manner, and – for a very wide variety of wave types – we typically find that when two waves meet, the amplitudes simply add (an observation we sometimes glorify as “the superposition principle”). But what if, at the point (in space and in time) where those two waves meet, one of the waves is at minimum in its cycle while the other is at a maximum? – The result, called “destructive interference” is still describable as simple addition of the two wave amplitudes (with crests counting as positive amplitude and troughs counting as negative amplitude). To determine whether CONSTRUCTIVE or DESTRUCTIVE interference will occur at any particular point of measurement on our observation screen, we need to consider the effects of any time lags that might accrue along different paths to the same point of observation.

Keep in mind that we use sinusoidal functions to describe simple waves, and that these expect an angle as their argument (also called a “phase angle”); the result is that time lags can also be expressed as a shift in the overall magnitude of the argument, the phase angle.
By explicitly considering the time lags (or phase shifts) that are associated with different paths coming from different parts of the aperture, we can show that the locations of the **MINIMA** *(i.e., the “darkest spots”)* in Figure 1 are analytically given by the Equation 1 below, and the **intensity** is described by Equation 2:

\[
I = I_o \left[ \frac{\frac{\pi a \sin \theta}{\lambda}}{\frac{\pi a \sin \theta}{\lambda}} \right]^2
\]

where \( m \) is an integer “index” used to count which of the minima we are referring to *(e.g., the two minima closest to the central fringe have an index of 1, the second closest have an index of 2, and so on).* Here, \( I_o \) is the intensity of the central fringe, \( \lambda \) is the wavelength of the light diffracted, \( a \) is the **width of the slit**, and \( \theta \) is the angle *(with respect to the optic axis)* made by a ray emanating from the slit and traveling to the point of measurement on the observing screen.

When the angle of diffraction, \( \theta \), is small, we can make a **small angle approximation**:

\[
\sin \theta = \theta = \tan \theta = \frac{y}{L}
\]

![Figure 4](image)

In the limit of small angles, our predictions, Equations (1) and (2), become:

\[
m\lambda = a \frac{y}{L}
\]

\[
I = I_o \left[ \frac{\frac{\pi ay}{\lambda L}}{\frac{\pi ay}{\lambda L}} \right]^2
\]

Place the slide with a **single slit** aperture (Slide 9165-A) in the location labeled “Slit” in Figure 1. Adjust its position to optimize the brightness of the resulting diffraction pattern. Sketch the observed diffraction pattern.

From fitting your observations to the predictions above, you can extract an experimental measurement of your laser’s wavelength:

Width of slit = ______________
\[ L \pm dL = \] 

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<thead>
<tr>
<th>( m )</th>
<th>( y \pm dy )</th>
<th>( \lambda_{\text{experimental}} )</th>
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Which order provides the more precise measure of the laser’s wavelength, \( m = 1 \) or \( m = 3 \)? Why?

Repeat this experiment with another single slit aperture from Slide 9165-A:

\[ \text{Width of slit} = \] 
\[ L \pm dL = \]

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Which slit provides the more accurate measure of the laser’s wavelength, the larger slit or the smaller one?
**Part 3: A single strand of hair**

In the next part of this lab, use your measured wavelength from part 2 and the diffraction pattern that results from placing a single strand of hair in the beam (instead of the slit) to measure the width of that strand of hair very precisely. If the strand of hair is oriented along the same direction that your slit aperture was, then all you’ve done by changing from the slit to the hair is to swap what was opaque in the diffracting plane with what was transparent. On the basis of this symmetry you can expect that the resulting pattern on your observing screen will be much like the pattern observed in Part 2, but with a reversal of dark and bright areas.

\[
\lambda \pm d\lambda = \text{__________}
\]
\[
L \pm dL = \text{__________}
\]

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What should you report as your experimental result for the diameter of the hair?

If you used a thinner strand of hair, how would that change the spacings between minima in the diffraction pattern?

**Questions**

1. In this experiment, we used a helium-neon laser, with \( \lambda = 632.8 \) nm. How narrow should your slit be, if you only want the first maxima to fit on the screen or sheet of paper you used for viewing the pattern?
2. Would a violet laser, $\lambda = 400$ nm, improve the accuracy of the measurement that you made on the wavelength? Why or why not?
Initiative

Conclusions