Background
Last time, we discussed the (topsy-turvy) fact that when light passes through a lens, a smaller focal spot results when a larger beam is incident upon the lens, as well as the fact that this phenomenon, like other basic laws of optics, is a result of interference between many different light rays (waves!). Practically, the upshot was that, in order to achieve the smallest spot size for high-resolution microscopy (or for a variety of clinical treatments), we need a large-diameter lens, in order to accept a large-diameter beam. The key result was that we now think of the transmitted light (as well as the nearby dark regions!) in terms of interference, a really fascinating phenomenon that we will systematically explore today, in the simplest cases we could think of.

Part 1: Double Slit
The experimental setup is the same as last time, shown in Figure 1. A monochromatic Helium-Neon laser, having a known wavelength of $\lambda = 632.8 \text{ nm}$, is positioned on the short bench. The laser should be positioned such that it points to the center of the screen. Objects causing diffraction will be positioned just a few centimeters from the laser. To get the greatest resolution, the (measured) distance between the object and the observing screen, $L$, should be as large as possible.

![Figure 1](image)

Today, instead of using a slide with the single slit aperture, you will examine what ensues when the laser is incident upon a slide containing a pair of very closely spaced slits (Slide 9165-B). Unlike last week, we will now measure the spacing between the MAXIMA in the intensity of the transmitted light.

Now that there are multiple slits, the spacing for the MAXIMA are given by Equation 1:

$$m \lambda = ds \sin \theta$$

where $m$ is the number of the maxima: the intensity peak nearest the central bright spot (or “fringe”) has $m = 1$, the second-nearest intensity peak is $m = 2$, and so on. Here, $\lambda$ is the wavelength of the light diffracted, $d$ is the distance BETWEEN the slits, $\theta$ is the angle from the slit to the measurement with respect to the location of the central fringe.
If the angle of diffraction, $\theta$, is small, we can make a small angle approximation:

$$\sin \theta \approx \theta = \tan \theta = \frac{y}{L}$$

Figure 2

In the limit of small angles, Equation (1), predicts intensity **MAXIMA** at locations given by:

$$m\lambda = d\frac{y}{L} \quad \text{(Double-Slit Intensity Maxima)} \quad (2)$$

Now, … before you start thinking that this is just what we did last time in lab, let’s look a bit more carefully at what we saw when we had a single slit aperture. In that case, we had found that there would be **MINIMA** in the intensity at positions given by:

$$m\lambda = a\frac{y}{L} \quad \text{(Single-Slit Intensity Minima)} \quad (3)$$

where *a* is the width of the slit.

Now, if you solve these equations for the expected locations, $y$, of these bright or dark spots, you see that there is a reciprocal relationship between $y$ and $d$ in Equation (2) and a similar reciprocal relationship between $y$ and $a$ in Equation (3). The upshot is that if $a << d$, then the single-slit minima will be much further from the optic axis than the spacing between the double-slit maxima. In that case, the intensity pattern we predict near the optic axis will be entirely determined by the interference between the two slits:

![Intensity pattern for a single slit](image)

**Fig. 3.** The expected double-slit interference pattern, when $a << d$.

That is, the intensity pattern associated with diffraction from one very narrow SINGLE slit should have a broad central maximum, and over the central portion of that pattern, the intensity is pretty uniform. So, if we do have $a << d$, then near the optic axis the observed pattern is just due to interference **between** the two slits.
On the other hand, let’s suppose value of the slit width, \( a \), is not quite so very, very narrow compared to the spacing between slits, \( d \). If we had only ONE of these slits, then (from last week) we expect the transmitted intensity profile to look like:

![Graph showing single-slit transmission pattern](image)

Fig. 4. A single-slit transmission pattern (as studied last time in lab).

Now, when there are TWO such slits, the central maximum will still contain the two-slit interference shown in Figure 3, but with an overall amplitude that is determined by the single-slit transmission shown in Figure 4.

![Graph showing two-slit interference pattern](image)

Fig. 5. In general, the amplitude of the two-slit interference pattern is determined by the single-slit transmission pattern.

In particular, note that if the single-slit diffraction pattern indicates that there will be no intensity at a particular point, as given by Equation (3), then that spot really will be dark even if Equation (2) shows that the waves coming from the two slits are in phase at that point. For this reason, some orders predicted by Equation (2) will appear to be “missing.”
**Procedure**

From fitting your observations to the prediction given by Equation (2), you can extract an experimental measurement of your laser’s wavelength:

Width of each slit, \( a = \) _____________
Spacing between slits, \( d = \) _____________

\[ L \pm dL = \] _______________

<table>
<thead>
<tr>
<th>( m )</th>
<th>( y \pm dy )</th>
<th>( \lambda_{\text{experimental}} )</th>
<th>% error</th>
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Part 2: Quadruple Slit

Before doing the experiment, qualitatively, what differences do you expect, in comparison to the double-slit aperture, as the number of slits increases? Please record your thoughts before proceeding:

Replace the slide with the double-slit aperture with a slide containing a quadruple-slit aperture (Slide 9165-C) and repeat the measurements made in the previous parts of this lab. From your sketches, record the relevant data in the table below:

<table>
<thead>
<tr>
<th>No. of slits =</th>
<th>Width of each slit, $a =$</th>
<th>Spacing between slits, $d =$</th>
<th>$L \pm dL =$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>$m$</th>
<th>$y \pm dy$</th>
<th>$\lambda_{\text{experimental}}$</th>
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Part 3: Diffraction Grating

Let’s now observe what happens when there are many, many, many slits. Now replace the slide with the quadruple-slit aperture with a slide containing a diffraction grating (e.g., Slide 9127), and examine what happens as you move your observing screen closer and closer to the laser.

For whatever $L$-value you decide is best for your measurements, sketch what you observe below. Be sure to include any relevant measurement values.

Is the small angle approximation still valid? If not, why?

Perform the appropriate calculation to determine the wavelength of the laser.

How do your previous results compare to this? Is there an improved accuracy and/or precision when using the diffraction grating? If so, why?
Part 4: *A two-dimensional diffraction pattern*

Leave your beam expander in place. Replace the diffraction grating with the large “Aperture Mask” (Slide 9139), placed on the far side of your component carrier, and make sure that the opening is reasonably centered on the beam. On the *other* side of that same component carrier, place a **two-dimensional** slit pattern (slide 9165-D) and observe the diffraction pattern produced. The observed pattern can be recorded below.

Can you explain the observed pattern from what you have already done in this lab? The following hints may help.

- Light obeys the **principle of superposition**.
- The two-dimensional slit pattern is the **superposition** of a number of one-dimensional slit patterns.
Initiative

*Possible ideas:*

1. Discuss how you could use the procedures in used in this lab to determine the structure of, say, a gamma globulin molecule? or DNA? or crystals of crystallite? Could you use a 632.8 nm laser? Sample calculations would make such an initiative complete.

2. You can supplement the required measurements with additional measurements on sections of the supplied slides that you have not yet used.

Conclusions