Jones Matrices:

The Jones Matrix formalism is a tool in handling problems dealing with the polarization of light. It takes advantage of the fact that a simple $2 \times 1$ matrix can be used represent the polarization state of a plane wave.

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

where $E_x$ and $E_y$ are complex numbers, encoding both amplitude and phase. Furthermore, the action upon that polarization state, by various optical components, can be simply represented by operating upon the input polarization state with appropriate $2 \times 2$ matrices. The resulting $2 \times 1$ matrix, called a “Jones vector,” represents the final state (polarization direction, amplitude, and any phase change) of the beam after passing through the optical component.

Table 1. A few commonly encountered Jones matrices

<table>
<thead>
<tr>
<th>Optical Element</th>
<th>Jones Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Polarizer</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>Vertical Polarizer</td>
<td>$\begin{bmatrix} 0 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>Linear polarizer, transmission axis at $\theta$ w.r.t. horizontal</td>
<td>$\begin{bmatrix} \cos^2 \theta &amp; \cos \theta \sin \theta \ \cos \theta \sin \theta &amp; \sin^2 \theta \end{bmatrix}$</td>
</tr>
<tr>
<td>Quarter-wave plate, fast axis at $\pm 45^\circ$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; \mp i \ \mp i &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>Quarter-wave plate, fast axis at $\theta$ w.r.t. horizontal</td>
<td>$\begin{bmatrix} \cos^2 \theta + i \sin^2 \theta &amp; (1 - i) \sin \theta \cos \theta \ (1 - i) \sin \theta \cos \theta &amp; \sin^2 \theta + i \cos^2 \theta \end{bmatrix}$</td>
</tr>
<tr>
<td>Half-wave plate, fast axis at $\theta$ w.r.t. horizontal</td>
<td>$\begin{bmatrix} \cos 2\theta &amp; \sin 2\theta \ \sin 2\theta &amp; -\cos 2\theta \end{bmatrix}$</td>
</tr>
</tbody>
</table>

For introducing the Jones Matrix formalism, you are to read Section 2.1 - 2.4 from our PHYS 317 text, *Quantum Mechanics: theory & experiment*, by Mark Beck, on the “Classical Description of Polarization” (also available in the library and the lab).

Also read Peatross & Ware Section 6.1 - 6.6

Expect a *QUIZ!*
Computational software such as *Mathematica* simplifies handling vectors and matrices, as documented in the built-in help or, *e.g.*, on the Wolfram website. A vector (such as a Jones vector, describing a polarization state) can be represented as:

\[
a = \{ \ a_1, \ a_2 \ \}
\]

A Jones matrix (*e.g.* one describing the action of an optical element) is represented by:

\[
b = \{ \ \{b_{11}, \ b_{12}\}, \ \{b_{21}, \ b_{22}\} \ \}
\]

Finally, for multiplication between matrices and vectors or matrices and other matrices, you need only type a period between the two:

\[b \cdot a\]

I strongly suggest that you make your matrices “look” more like matrices: in *Mathematica*, the command `MatrixForm[ ]` is used to display the result in 2D matrix form. You should check out the *Palettes* menu, which includes *Basic Math Assistant*, which provides a 2D template for matrix entry. Also note that you can simply highlight rows in the template and hit delete to remove ones you don’t want (*e.g.*, to make a column vector).