# Jones Matrices:

The Jones Matrix formalism is a tool in handling problems dealing with the polarization of light. It takes advantage of the fact that a simple $2 \times 1$ matrix can be used represent the polarization state of a plane wave.

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

where $E_x$ and $E_y$ are complex numbers, *encoding both amplitude and phase*. Furthermore, the *action upon* that polarization state, by various optical components, can be simply represented by operating upon the input polarization state with appropriate $2 \times 2$ matrices. The resulting $2 \times 1$ matrix, called a "Jones vector," represents the final state (*polarization direction, amplitude, and any phase change*) of the beam after passing through the optical component.

Table 1. A few commonly encountered Jones matrices

| **Optical Element** | **Jones Matrix** |
|---|---|
| Horizontal Polarizer | $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ |
| Vertical Polarizer | $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ |
| Linear polarizer, transmission axis at $\theta$ w.r.t. horizontal | $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ |
| Quarter-wave plate, fast axis at ±45° | $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \mp i \\ \mp i & 1 \end{bmatrix}$ |
| Quarter-wave plate, fast axis at $\theta$ w.r.t. horizontal | $\begin{bmatrix} \cos^2\theta + i\sin^2\theta & (1-i)\sin\theta\cos\theta \\ (1-i)\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{bmatrix}$ |
| Half-wave plate, fast axis at $\theta$ w.r.t. horizontal | $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ |

For introducing the Jones Matrix formalism, you are to read [Section 2.1 - 2.4](#) from our PHYS 317 text, *Quantum Mechanics: theory & experiment*, by Mark Beck, on the "Classical Description of Polarization" (also available in the library and the lab).

Also read Peatross & Ware [Section 6.1 - 6.6](#)

Expect a *QUIZ!*

Computational software such as *Mathematica* simplifies handling vectors and matrices, as documented in the built-in help or, *e.g.*, on the Wolfram website. A vector (such as a Jones **vector**, describing a polarization state) can be represented as:

```
a = { a1, a2 }
```

A Jones **matrix** (*e.g.* one describing the action of an optical element) is represented by:

```
b = { {b11, b12}, {b21, b22} }
```

Finally, for multiplication between matrices and vectors or matrices and other matrices, you need only type a period between the two:

```
b.a
```

I strongly suggest that you make your matrices "*look*" more like matrices: in *Mathematica*, the command MatrixForm[ ] is used to display the result in 2D matrix form. You should check out the **Palettes** menu, which includes **Basic Math Assistant**, which provides a 2D template for matrix entry. Also note that you can simply highlight rows in the template and hit delete to remove ones you don't want (*e.g.*, to make a column vector).