Your First *LabVIEW* Lab: The Stefan-Boltzmann Law (at high temperatures)

Purpose: To find the typographical *error* in one of the following formulae.

We've shown that you can use **Bose-Einstein statistics** to derive Planck's radiation law as well as the *Stefan-Boltzmann law*, which says that the radiant power emitted from a hot body is proportional to the ***fourth*** power of its temperature. Since Planck's radiation law was *historically* considered to be a 'proof' that energy is quantized, a test of the Stefan-Boltzmann law is not only a test of **Bose-Einstein statistics** but also, to some degree, of energy quantization *even though it is a continuous distribution!*

In your version of the experiment, the temperature of the radiating body is determined from its electrical resistance. Electrical resistance, too, is something best explained via quantum theory! (Quantum physics is essential!)

Equipment: Radiation sensor and support
Stefan-Boltzmann Lamp, Heat Shield
Glass Thermometer
Analog Ohmmeter, Ammeter, 0 - **3 A**, which is **beyond** your **MAX** DAQ limits
Analog Power supply, 0 - **13 V**, 0 - 3 A, **beyond** your **MAX** DAQ limits
Analog Voltmeter, 0 - **13 V**, **beyond MAX** DAQ limits (though *happiness* is a **voltage divider**),
DAQ-based Digital Millivoltmeter, with
your own *LabVIEW* program for datalogging **(write this yourself!!!)**

Theory:

Josef Stefan found *empirically* and Boltzmann then supplied the theoretical explanation that the total power radiated by an object is given by:

$$P = \sigma A e T^4 \qquad (1)$$

where P is the radiated power in watts; T is the absolute temperature of the object; A is the total surface area of the object in meters; e, the emissivity, is dimensionless and equal to unity for a perfectly black object; and the Stefan-Boltzmann constant, σ, is equal to:

$$\sigma = \frac{\pi^2}{60}\frac{k_B^4}{\hbar^3 c^2} = 5.67051 \pm 0.00019 \times 10^{-8}\,\frac{\text{Watts}}{\text{m}^2\text{K}^4}.$$

Notice that the *intensity* (power per unit area) of thermal radiation coming from an ideal black body would be simply:

$$\frac{P}{A} = \sigma T^4 \qquad (2)$$

- What is the radiated power from a black body at zero Kelvin?

- With what intensity does a light bulb filament at room temperature, 21 °C, radiate?

<u>EQUIPMENT</u>

<u>Stefan-Boltzmann Lamp</u>:

By adjusting the power sent in (13 Volts Maximum, except for ***brief measurement intervals***), filament temperatures up to approximately 3000 °C can be obtained. Using a high temperature lamp simplifies the analysis because, in the high temperature limit, the fourth power of the <u>*ambient environment*</u> should become negligible compared to the fourth power of the high temperature of the lamp filament.

Also, <u>*when properly oriented*</u> (pay attention!), the filament provides a good approximation to a *point source* of thermal radiation. Thus, the inverse square law may be incorporated into analysis of the data.

Most of the thermal energy from a lamp comes from the filament, and the filament can be considered an electrical resistor. Resistance of a conductor depends upon its temperature, so we will use the resistance as a measure of the temperature of the filament.

From:

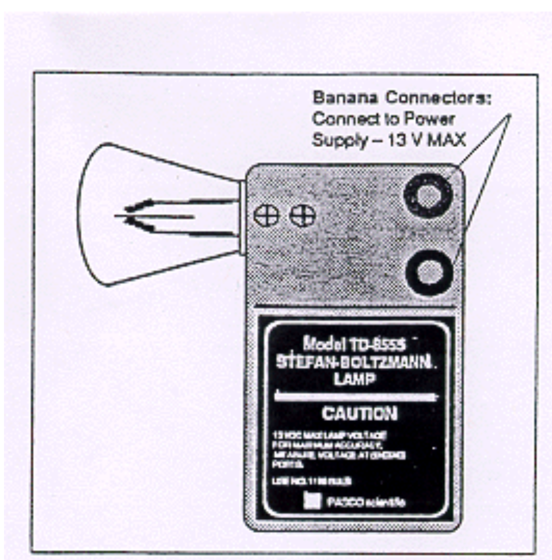
$$R = R_0\left[1+\alpha(T-T_0)\right] \qquad (3)$$

<u>you</u> <u>*should be able to show*</u>:

$$T = \frac{(R-R_0)}{\alpha R_0} - T_0 \qquad (4)$$

where R_0 is the filament resistance at a known temperature, T_0 (say, room temperature, as measured with your glass thermometer). R is the resistance at temperature T and α is the temperature coefficient of resistance for the particular filament. [$\alpha_{(\text{tungsten})} \sim 4.5 \times 10^{-3}\ \text{K}^{-1}$]

When the filament is *OFF* and disconnected, its resistance can be measured with an ohmmeter. When the filament is *ON*, its resistance can *INSTEAD* be determined by using Ohm's law, $R = V/I$, where V = the voltage across the filament, and I = current through the filament.



Stefan-Boltzmann Lamp

IMPORTANT: The voltage into the lamp should not exceed 13 V **for any *<u>significant</u>* period of time**. Such "high voltages" will shorten the lifetime of the filament.

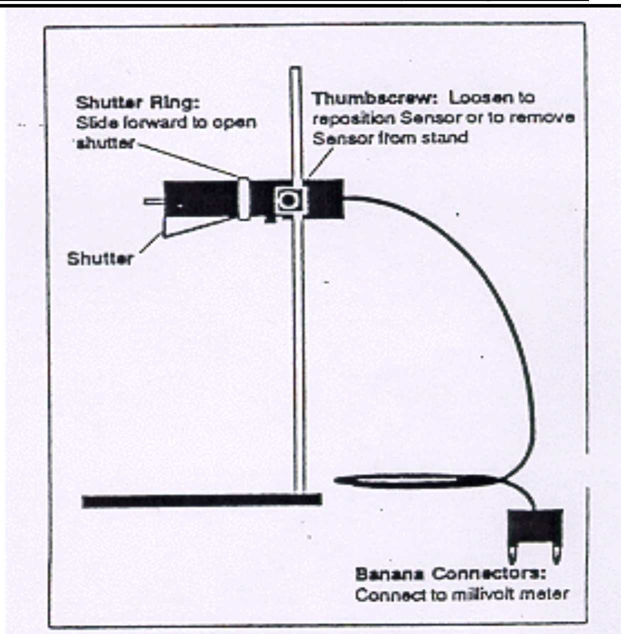
Radiation Sensor:

The PASCO *TD 8553* Radiation Sensor measures the relative intensities of incident thermal radiation. The sensing element, a miniature thermopile, produces a voltage proportional to the ***power*** contained within the incident radiation. The spectral response of the thermopile is essentially flat in the infrared region (from 0.5 to 40 micrometers). Q: Why does this matter?

The output voltages produced by the illuminated thermopile range from the microvolt range up to around 100 millivolts. Q: What is the ***resolution*** of your DAQ?

The Sensor can be hand held or mounted on its stand for more accurate positioning. A spring-clip shutter is opened and closed by sliding the shutter ring forward or back. During experiments the **shutter should be closed when measurements are not actively being taken**. This helps reduce temperature shifts in the thermopile reference junction that can cause the sensor response to drift.
Q: How can an experimentalist deal with *drift*?

*NOTE: When opening and closing the shutter, you might inadvertently change the sensor position. Therefore, *for any experiments in which the sensor position is critical*, a small sheet of opaque insulating foam has been provided. **Place this heat shield in front of the sensor when measurements are not actively being taken.**



Radiation Sensor

The two posts extending from the front end of the Sensor protect the thermopile and also provide a reference for positioning the sensor a repeatable distance from a radiation source.

Specifications:

Temperature Range:	- 65 to 85 °C.
Maximum Incident Power:	0.1 Watts/cm^2
Spectral Response:	Flat from 0.5 to 40 micrometers
Signal Output:	Linear from 10^{-6} to 10^{-1} Watts/cm^2.

Procedure:

CAUTION: Do not let the lamp filament voltage exceed 13V.
High voltages will burn out the filament or shorten its lifetime.

1. **The manual for any particular DAQ (*e.g.*, the National Instruments PCI-6251), can be found online. *Look up* the specs for your DAQ: Maximum Sampling Rate, Input Impedance for Analog Voltage Inputs, and Output Impedance for Analog Outputs.**

The radiation sensor should be at the ***same height*** as the filament and about 6 cm away from it. The entrance angle of the sensor should include no other close objects. The sensor's reading is proportional to P.

2. Get R_0 and T_0 **before** making circuit connections:

3. With the power supply set for *low* voltage, turn it on and obtain V and I readings as suggested in the data table. At each V, also measure the "*Rad*" (radiation sensor reading, in mV). **Make each reading *quickly***. Between readings insert the reflecting heat shield between the lamp ant sensor so that the sensor temperature stays relatively constant. (Coordinate yourselves.)

4. Complete the data table: Get R (ohms) of the filament from V and I.
Get T(K) from Eq. (4), and then calculate T^4 (K^4).

DATA			CALCULATIONS				
V (Volts)	I (Amps)	*Rad.* (mV)	R (Ohms)	T (K)	T^4 (K^4)		
1.00							
2.00							
3.00							
4.00							
5.00							
6.00							
7.00							
8.00							
9.00							
10.00							
11.00							
12.00							

$T_0 =$ ____________. $R_0 =$ ____________.

<u>Analysis</u>:

If you plot *Rad*. (mV) vs. T^4 you may see departure from a linear relationship, if the experimental exponent is not *exactly* four, and you might be left wondering whether there might exist a T^n relationship to P. So, what's an alternative plot you might make?

That is, our original hypothesis was that $P[Rad(\mathrm{mV})] = bT^4$. Let's relax this somewhat, to a *generic* "power law:"

$$P[Rad(\mathrm{mV})] = bT^n.$$

Take the logarithm of each side, and we see:

$$\ln(P) = \ln(b) + n\ln(T) \text{ (This should not surprise you!)}$$

$$\ln(P) = n\ln(T) + \ln(b)$$

- which looks like:

$$y = mx + b$$

So, if you plot ln[*Rad*(mV)] vs. ln T, and *if a power law relationship exists*, then you will not only see the linear relationship between ln P and ln T. but the slope, m, measured from the plot, will tell you what your experimental value of the exponent is.

Things to consider:

- What is the meaning of any non-linearity of any plot you construct?

- Is the filament black? If the filament was "half-black" (that is, $e = 0.5$), would that alter the intended purpose of the experiment?

- Will your data and results be affected if the relative *position* of lamp and sensor changes during the experiment?

- Between readings what side of the heat shield should face the lamp?

- Did *you* actually show that Eq. (4) followed from Eq. (3)?

Questions:

1. If the surface of the Sun has a temperature of about 5800K and a radius of 6.96×10^8 m - and is considered to be a black body, then what is the solar irradiance at the *top* of the Earth's atmosphere in Watts/m^2? [Does your answer agree with the numbers used for homework problems in PHYS 106? (Yes, I do expect you to CHECK!)]

2. At Earth's *surface*, solar flux is about 1×10^3 Watt/m^2. If a black sheet of paper faces the sun what will become the equilibrium (constant) temperature of the paper? [Assume the back side of the paper is perfectly insulated so that the only heat loss is by blackbody radiation from the top surface.] What is the meaning of "equilibrium" in this question and the next?

3. From your answer to #1 and the ideas of #2 estimate an equilibrium temperature for the Earth. [Assume it also radiates like a blackbody at constant temperature.]

4. If the spectral power radiated per unit area by a blackbody at constant temperature T in wavelength interval $\delta\lambda$ is given by $I(\lambda,T)\delta\lambda$, in units of Watts/m^2, how would you find the total power radiated per unit area at this T, over all wavelengths?

Other discussion points:

A) The manual suggests that 13V might, for the lamp as provided by the manufacturer, yield something close to 3000K. Does it? Why might it not? If the maximum observed temperature is less than 3000K would the warning not to exceed 13V still apply? Why or why not?

B) Whenever you want to make a *precision* measurement of any resistance of 1000Ω or less, it is suggested that you use the **Kelvin "4-Probe" method** for determining resistance. Why?

C) By fitting the rise time and decay time of the thermopile signal as you remove or replace the heat shield, you can extract the characteristic *time constants* for the detector. Are these independent of the incident power? Given these time scales, is it possible for you to use this detector to observe the 60Hz flicker of the fluorescent room lights? – How do you expect the contribution due to the fluorescent room lights to show up? – How can you justify any *processing* performed on your raw signal?

D) If your personal goal was to analyze every single aspect of your experiment that you possibly could, given the equipment available to you, what else could you do? Would reading Piazza Note @161 help? [Why, yes, yes it would!]

PLEASE include suggestions for improvement of your experimental method/analysis:

Use this space for a compelling display of initiative and insight:

***Specific* Conclusions and Critical Analysis:**