Spectra & the Origin of Quantum Physics

The Prequel: Taking the temperature of stars!
When something is hot enough, it glows. Why? – Well, if the charged objects that make up matter are thermally agitated, that means that the field lines associated with those charged objects must also be “jiggling” … and so it seems reasonable to expect that electromagnetic waves would radiate from any hot object. For objects at room temperature or a bit warmer (like you), this thermal radiation is mostly outside of the visible portion of the spectrum (peaking, instead) in the infrared. Even for something cooled to the temperature of liquid helium, there is still detectable thermal radiation (though the peak is then in the microwave portion of the spectrum). On the other hand, as we heat something to temperatures well above room temperature, significant radiation appears in the visible portion of the spectrum. All of this allows for remote (spectral) assessment of temperature, which plays an important role in our understanding of extreme environments, whether involving molten metal at a steel plant in Pittsburgh, or a research fusion reactor.

**Thermal Radiation Color Chart** (Save, in your lab notebook, for future reference):

<table>
<thead>
<tr>
<th>Color Description</th>
<th>°F</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dull Red &quot;visible at night&quot;</td>
<td>750</td>
<td>400</td>
</tr>
<tr>
<td>Red &quot;visible at twilight&quot;</td>
<td>885</td>
<td>475</td>
</tr>
<tr>
<td>Red &quot;visible in daylight&quot;</td>
<td>975</td>
<td>525</td>
</tr>
<tr>
<td>Red &quot;visible in sunlight&quot;</td>
<td>1075</td>
<td>590</td>
</tr>
<tr>
<td>Dark Red</td>
<td>1290</td>
<td>700</td>
</tr>
<tr>
<td>Dull Cherry Red</td>
<td>1475</td>
<td>800</td>
</tr>
<tr>
<td>Cherry Red</td>
<td>1650</td>
<td>900</td>
</tr>
<tr>
<td>Bright Cherry Red</td>
<td>1830</td>
<td>1000</td>
</tr>
<tr>
<td>Orange Red</td>
<td>2010</td>
<td>1100</td>
</tr>
<tr>
<td>Orange-Yellow</td>
<td>2190</td>
<td>1200</td>
</tr>
<tr>
<td>Yellow-White</td>
<td>2370</td>
<td>1300</td>
</tr>
<tr>
<td>White</td>
<td>2550</td>
<td>1400</td>
</tr>
<tr>
<td>Brilliant White</td>
<td>2730</td>
<td>1500</td>
</tr>
<tr>
<td>Blue White</td>
<td>2900</td>
<td>1600</td>
</tr>
</tbody>
</table>

(Perhaps someone could help me convert this to color printout.)

**Notable Temperatures:**

- Acetylene Flame = 4080 °F = 2250 °C
- Induction Furnace = 5450 °F = 3000 °C
- Electric Arc Furnace = 7200 °F = 4000 °C

Your instructor has used a “dimmer” to turn on a simple light bulb. Given the chart above, how hot would you estimate the filament temperature to be (roughly)?

Hold a diffraction grating in front of your eye. When you look at the filament straight through the diffraction grating, you should see the same color you saw without the grating. This is because each of the (distinct) diffraction patterns associated with each of the emitted wavelengths has a “zero-order” ($m = 0$) maximum located on the optic axis.
Without moving the diffraction grating, just turn your eye to one side, and you should see that the **first-order** \((m = 1)\) diffraction maxima for each of the emitted wavelengths is in a slightly different position: in this way, we have “dispersed” the emitted radiation into its component wavelengths. If you now turn your eye to the other side, you can view the \(m = -1\) spectrum. – **Write into your lab notebook some comments, e.g., comparing the \(m = 1\) and \(m = -1\) spectra.**

In the year 1900, Lord Kelvin gave a speech to the Royal Institution, aimed at identifying the most vexing problems in physics. At the time, physicists were (rightly) extremely proud of the wide-ranging successes of Newtonian methods. Kelvin, however, pointed to “two clouds on the horizon,” which had resisted all explanation thus far. These two difficult issues were: How does light propagate? And how do we explain (even qualitatively) the relative intensities of thermal radiation as a function of wavelength? – As it turns out, neither of these (seemingly innocuous) questions (nor quite a few others) **could** be answered without a **revolution** in what we take as fundamental starting points in science. Indeed, Max Planck’s solution to the problem of thermal radiation, and the (many) key follow-ups by Albert Einstein (named in 2000 as Time Magazine’s "**Person of the Century**") shook the very foundations of science. These problems led to the origins of Quantum Mechanics.

Examine how the spectrum of thermal radiation changes as the light is made dimmer or brighter. **Record your observations into your lab notebook.**

**Part 1: Characteristic Spectra**

In astronomy, the (relatively cool) surface temperature of stars ranks with mass and diameter as one of the three quantities basic to an understanding of the star’s internal structure and of the associated photospheric activity. The basic curve that Max Planck derived for **any** hot object even works when applied to stars but, that said, there are some very sharp spikes and gasp in the spectra we observe when looking at stars, which turn out to tell us about what elements are present.

In fact, when we look at the light emitted by any pure, elemental gas, instead of a continuous spectrum, as one might expect, only discrete (characteristic) wavelengths of radiation are observed. This, too, reflects the fact that fundamental microscopic processes in nature are quantized, one of the crowning achievements of 20th century physics.

**Theory:**

1) Even before modern, quantum theory was developed, there was a very simple, “semi-classical” \((i.e., \textbf{WRONG!!!})\) theory by Neils Bohr was able to predict which wavelengths would be emitted by any gas of atoms containing a single electron. The Bohr model assumes \((\textbf{INCORRECTLY!!!})\) that the electron exists as a localized particle, in a classical orbit around the nucleus (akin to a planet orbiting a star). In attempting to fit his model to the observed characteristic spectra of emitted light, Bohr first asked whether the allowed “so-called orbits” might simply have radii that were multiples of some fundamental unit. That didn’t give a result that fit the data. Bohr also considered whether the allowed “orbits” might simply have energies that were multiples of some fundamental unit. That didn’t give a result that fit the data, either. However, when he tried assuming that (for some reason) the allowed “orbits” might all have (or at least are somehow capable of transferring) **angular momenta** that were multiples of some fundamental unit, which turned out to be \(\hbar = h/2\pi\), he found that his model fit the observed data very, very well (and even correctly **predicted** emitted wavelengths outside of the visible portion of the electromagnetic spectrum, which had not yet been observed, but which were observed soon afterwards).
Bohr’s basic idea was that whenever an electron makes a transition from one of the larger allowed “orbits” to a smaller one, a discrete amount of energy will be released. This overly simplified theory (as well as modern Quantum Mechanics) predicts that, for hydrogen, the only transitions that yield visible light are those associated with transitions from a higher level down to level 2. The wavelength, $\lambda$, of light emitted during the transition from a level $n$ down to level 2 is given by the equation:

$$\frac{1}{\lambda} = \frac{e^4 \mu}{8\varepsilon_0^2 h^3 c} \left( \frac{1}{2^2} - \frac{1}{n^2} \right); \mu = \text{reduced mass} \tag{1}$$

$$= R_H \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \tag{2}$$

where $n$ is a positive integer and $R_H = (1.096775834 \times 10^7 \text{ m}^{-1}) = 10.96775834 \mu\text{m}^{-1}$ is called the Rydberg constant.

These results are precise for vacuum measurements only.

II) A diffraction grating sets up a diffraction pattern. As you know, the equation for the location of maximum intensities is:

$$m\lambda = d \sin(\theta) \tag{3}$$

Hence if the grating spacing, $d$, is known one can calculate the wavelength of light radiated from a substance, at least if you can measure the diffraction angle well. The diffraction grating spectrometer is designed for that very purpose: to make very precise measurements of the diffraction angle, $\theta$.

Procedure:
For transitions from $n = 3, 4, 5$ in Eq. (1), visible light is emitted from the hydrogen atoms. It is these emissions that we will study (the Balmer Series). Except for selecting your diffraction grating and aligning the source, the spectroscope is already set up properly, so you needn't make any other adjustments. So, begin simply by selecting your diffraction grating. If, for example, you select a grating with 15,000 lines/inch, you can find that $d = 16.933 \text{ Å}$. That is, the periodicity of the grading is found by inverting the number of lines per inch and converting from inches.

Your lab notebook should identify:

Grating selected = _________

$d = ______________________$

Once you’ve mounted your grating onto your spectrometer, qualitatively observe the diffraction pattern of the hydrogen radiation through the spectroscope. Notice that there are, indeed, discrete wavelengths of light coming from the gas and not a continuous spectrum of colors. Observe as many lines as possible in at least two orders. We are really only interested in the bright lines (red, blue-green, and violet); any faint, narrow, and closely spaced lines are not due to transitions in atomic hydrogen, but arise from molecules of hydrogen which may also be present in the tube. (Newer tubes have more atomic hydrogen and less molecular hydrogen.)

Sketch, in your lab notebook, your observed spectrum. Be sure to identify characteristics such as color, brightness, etc.
Having qualitatively verified the *discrete* nature of the spectrum (*i.e.*, the quantization of atomic radiation), let us now determine whether or not the wavelength of the radiation quantitatively agrees with the Bohr theory of the atom (Eq. 1). Measurements will be made on only the *first* order spectrum [(*m* = 1) in Eq. (3)]. Move the telescope to be collinear with the collimator tube, to look directly at the *m* = 0 image of the discharge tube rather than at any individual line of the spectrum. Note the reddish color of this central diffraction maximum. Adjust telescope *focus* until the edges of the slit are sharp and clear. Narrow the slit as much as possible (Why? This is worth puzzling over in your lab notebook a bit), but not so much that it becomes difficult to see. Move the telescope *left* until the *cross hair* is on one of the spectral lines, left of the center. Record the reading on the angle scale. Move the telescope to the same color line *right* of center, again recording the setting. For each spectral line, *one half the difference* between the two readings is the diffraction angle \( \theta \). The sort of table that might appear in your lab notebook is illustrated below:

<table>
<thead>
<tr>
<th>Color</th>
<th>( \theta_L ) (Left Angle)</th>
<th>( \theta_R ) (Right Angle)</th>
<th>( \theta = \frac{\theta_L - \theta_R}{2} ) (Mean Angle)</th>
<th>Exp. ( \lambda ) (Å)</th>
<th>Std. ( \lambda ) (Å) (air)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Violet (H(_g))</td>
<td></td>
<td></td>
<td></td>
<td>4340.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green (H(_b))</td>
<td></td>
<td></td>
<td></td>
<td>4861.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red  (H(_a))</td>
<td></td>
<td></td>
<td></td>
<td>6562.79</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With this equipment, the *experimental values should be less than 50 Å from the accepted value*. A little care in technique will achieve this (*e.g.*, avoiding any effects of mechanical “backlash”). If *not*, repeat the measurements *carefully*! (The accuracy in reading the angles ought to be 1.0 minute of arc, or so, depending upon the mini-spectrometer you’re using. Better resolution is possible with larger spectrometers) – From your measured wavelengths, calculate the Rydberg constant, \( R_H \). Remember that your measurements are air values, and - so far - have not taken into account the index of refraction of air. Since the index \( n_{\text{light}} \) is defined by \( n_{\text{light}} = c/v_{\text{air}} \), we get immediately \( \lambda_{\text{air}} = \lambda_{\text{vac}}/n_{\text{light}} \). Then according to Eq. (2) we see \( R_H \sim 1/\lambda \), which in turn means:

\[
R_{H_{\text{vac}}} = \frac{R_{H_{\text{air}}}}{n_{\text{light}}}
\]

Hence we can correct our calculated value for the Rydberg constant by dividing by the appropriate index of refraction for the given laboratory conditions. Use the CRC handbook to find corrections.

II. Your lab notebook should contain:

Rydberg constant from H\(_a\) ________.

from H\(_b\) ________.

from H\(_g\) ________.

Ave. \( R_H \) = ________ (air)  Index of refraction, \( n_{\text{light}} = ________ \).

Ave. \( R_H \) = ________ (vacuum)  % error = ________.

Accepted value \( R_H = 1.0967758 \times 10^7 \) m\(^{-1}\) (vacuum)
**Specific Conclusions and Critical Analysis:**

[Continue working to make your discussion of errors more specific and “professional” - that is, try to estimate the magnitude and direction of skew of those errors you suggest, and try to incorporate at least some numerical work on uncertainties]

**Next Steps:**

1. Compare the discrete characteristic line spectrum you’ve seen here to be emitted by an excited gas of atoms or molecules, to the **broader** spectrum emitted by a solid-state source, such as a Light-Emitting Diode (LED).
2. Demonstrate diffraction using microwaves, which have such a **long** wavelength that you can see the slits.
3. Move in the other direction, to x rays and gamma rays, which have extremely **short** wavelengths, and perform **spectroscopy** on their interaction with matter.
4. Compare your results using the light from an excited hydrogen vapor to what you get when you study the same source by using a grating with a different number of lines per inch.
5. Use a high-brightness mercury vapor source, which provides **many** spectral lines. Diffraction allows you to systematically select out individual frequencies, and “grey-scale” filters allow you to systematically vary their intensity, so as to test Einstein’s predictions regarding the PhotoElectric Effect, to prove the existence of the photon.