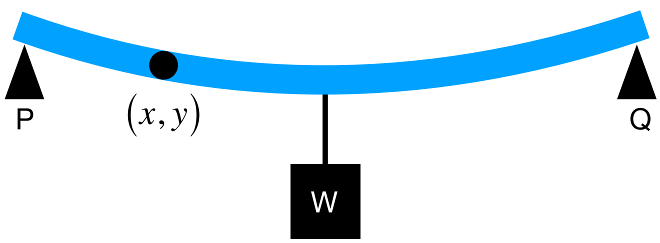
HW #8: Non-uniform Strain

AFTER working through the reading from Stephenson's "Mechanics and Properties of Matter," consider a

Three-Point Flexure Test:

At left is a **beam** oriented along the x-axis, of length ℓ, supported at fulcrum points.

Below, a weight, W, is added in the middle. There is an upward supporting force, P, at the left support, and another, Q, at the right support. As the weight of the beam acts at the center of mass, which is the where we add external weight, we can take W to be the total of the beam weight and the applied weight.



In equilibrium, the forces must cancel:

$$P + Q - W = 0$$

We will use the unusual notation adopted by Stephenson's text, calling the external torques, L. (Another unusual convention: engineering texts will sometimes call these the "bending moments.")

The equilibrium condition is then given by Stevenson's Eqn. 7.7, which states that the external torques must be balanced by the internal torques associated with the strain fields inside the material, which appear on the left side of the equilibrium condition:

$$EI_A \frac{d^2y}{dx^2} = L$$

The sign convention utilized in Stephenson's text is that the torque, L, about any point is positive whenever it promotes positive curvature in the beam. As the geometry given above (which corresponds to your next lab) is different from the geometries given in Stephenson, you have to explicitly write down those torques yourself:

A) Turn in your best attempt to **derive** that a measured mid-point deflection of y_{mid} will result when the elastic modulus is:

$$E = -\frac{\ell^3}{48 I_A \left(y_{\text{mid}} / W \right)}$$

Hint: before evaluation at the midpoint, where $x = \ell/2$, you should find a more general result describing the equilibrium deflection at any arbitrary x coordinate:

$$y = \frac{W}{EI_A}\left(\frac{x^3}{12} - \frac{x\ell^2}{16} \right)$$

B) Explicitly show that, for a beam with rectangular cross-section, of width b and thickness a, the second moment of area is:

$$I_A \equiv \int y^2 dA = \frac{ba^3}{12}$$