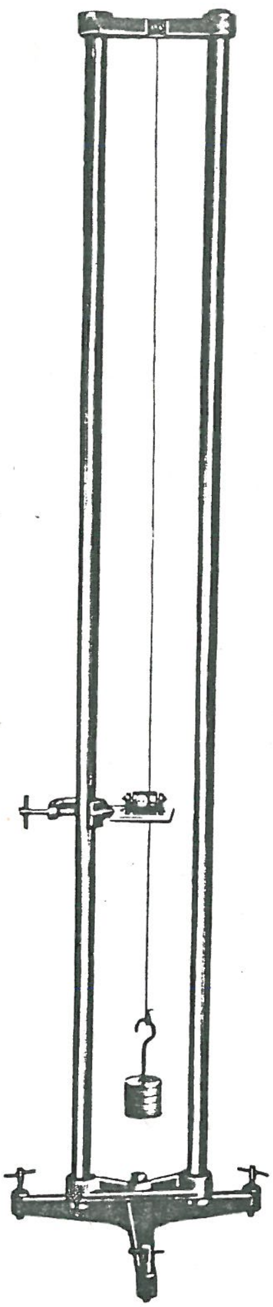
HW #6: Elastic Modulus

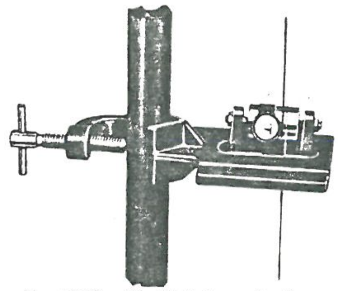
*1. – **The case of UNIFORM applied stress**:* Write up how this <u>*lab*</u> extracts the elastic modulus:

To ensure that you measure distances appropriately, re-write the attached document, making use of standard units (and the fact that, in the lab, your wire is wrapped around a cylindrical spool, ***rather than*** using the sort of mirror mount labeled as No. 3542 in the figure below).

INSTRUCTIONS FOR THE USE OF NO. 572 AND NO. 572D
YOUNG'S MODULUS APPARATUS AND NO. 572G FITCH LEVER ATTACHMENT



NO. 572D



No. 572G with Fitch Lever in Place.

No. 3542.

This apparatus is designed for the determination of Young's Modulus of a wire, one of the fundamental constants in mechanics. Two types of apparatus may be used, the only change being in the form of optical lever used. The experiment in its essentials consists of (1) providing a fine wire fastened at the upper end to a firm support; (2) a convenient hanger secured to the lower end of the wire on which equal weights may be successively placed to stretch the wire; and (3) a means of measuring accurately the stretch produced in the wire by each addition of weight. In this experiment two forms of optical lever are described for the measurement of this elongation.

<u>No. 572 Apparatus with Standard Optical Lever</u>

In this form of optical lever a small chuck is clamped to the wire at a convenient point and a platform which is fastened to one of the support rods provides the proper support for an optical lever, the single foot of which rests upon this chuck. The lever will be found to have a short base (2.5 cm) with pointed feet, the shortness of the lever providing a greater deflection of the spot of light, thereby increasing the accuracy of the observations over the usual type. The two feet opposite the single foot of the lever should rest in a groove provided in the platform so as to keep the optical lever in the same position. An ordinary reading telescope should be set up on the laboratory table approximately one meter's distance away, and a vertical scale attached to the telescope support. Locate the image of the scale in the telescope and so adjust the mirror of the optical lever that the portion of the scale observed in the telescope is approximately as far below the axis of the telescope as the deflection will ultimately go above during the process of adding weights. (The operator must bear in mind that the image of the scale will be reversed.) This prevents abnormal distortion of the angular displacement. Evidently, now, the operator is in position to add equal weights to the hanger and produce deflections in the telescope which can be measured with considerable accuracy. The starting weight used should be amply sufficient to remove all kinks from the wire before the actual observations are begun.

The experiment consists of the successive addition of equal weights within the limit of the capacity of the wire, say four or five 1-kilo weights, followed immediately by the successive removal of these weights, each time observing the deflection in the telescope. A very fine wire (steel) is sent with the apparatus and with this wire, units of 200 grams will be better than kilo weights. A reasonable time should be given between each succeesive observation for the changing system to come to absolute equilibrium, say a minute or so between each observation. This allows the wire to reach its maximum stretch without producing a fatigue in the wire which might affect the observations. Care should be taken to note whether the final scale deflection after all of the original weights have been removed is essentially the same as at the start. If this is not true, two things are possible, one that the chuck holding the upper end of the wire is not tight enough and is slipping on the wire, and the other that too many weights have been attached, thus exceeding the elastic limit of the wire itself. If such a discrepancy is noted it is more likely due to the slip on the wire of the chuck and this should be tightened so as to stop the slipping.

Measure with a meter stick the distance from the mirror to the scale and record this as the distance D. Measure accurately the length of the lever by pressing the points down on a sheet of paper, drawing a fine line between the two feet about which the lever turned (the feet which were set in the groove) and measuring the perpendicular distance from this line to the point of the third foot. This distance should be called ℓ . Now measure the length of the wire from the lower face of the chuck holding the upper end of the wire to the upper face of the chuck upon which the foot of the optical lever rests. This should be called L. Measure also the diameter of the wire by means of a micrometer caliper expressing this in centimeters. Take one-half and call it r . Evidently the angle e/ℓ in which e is the actual elongation of the wire for one weight is equal to one-half the angle d/D in which d is the mean deflection and D the distance between the scale and the mirror.

(The spot of light goes over twice the angle that the mirror actually moves which accounts for the factor 2). Young's Modulus may therefore be calculated from the formula

$$Y = \frac{2 \times 980\ W\ L\ D}{\pi\ r^2\ d\ \ell}$$

in which W is the unit weight used for the observations expressed in grams; 980 is the constant of gravitation; π has the usual value, and the other values are given above, all expressed in centimeters.

As Young's Modulus is defined as "the ratio between the force per unit cross section of the wire, and the elongation per unit length" this will be a very large quantity. It is usual, therefore, to express the result as a numeral of two digits times a factor of 10^{11}. The accepted value for steel is about 12, 4 x 10^{11}.

No. 572D and No. 572G Fitch Optical Lever Attachment

Professor A. L. Fitch of the University of Maine has designed a modification as shown in the illustration which makes use of a mounted cylinder around which the wire is wrapped once, this cylinder being mounted between the two cone bearings and free to turn. A mirror is fastened to a clip which fits over the cylinder and thus provides the deflections in the same manner as the regular optical lever. Obviously the deflections produced will indicate the elongations of the wire in the same manner as above, the radius of the cylinder (its diameter being obtained by means of a micrometer caliper) being the length of the lever. In every other particular the details of the experiment are the same as deecribed for the other type of lever.

On account of the short optical lever (approximately one centimeter) deflections produced on the scale will be greater than the previously described apparatus. This should produce a higher accuracy, particularly in view of the fact that the length of the optical lever (the radius of the cylinder) can be more accurately measured with a micrometer caliper than in the case of the point form of optical lever, although this in turn is still more accurate than the ordinary form. As before, sufficient time should be given for the system to come into equilibrium before taking observations.