HW #12: Band Structure

1. – ***Density of states for ELECTRONS in 2D solids****:*

Hofmann Ch 6 walks you though calculating the density of states for a free electron gas in a 3D solid, ending up on page 97 with Equation 6.13:

$$g(E) = \frac{dN}{dE} = \left[\frac{V}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} \right] E^{1/2}$$

A) Following a similar procedure, show that the density of states for a free electron gas in a 2D solid is independent of the energy.

B) How would the density of states depend on the energy if the electron dispersion were *linear* instead of *quadratic*? (That is, if $E(k) \propto k$ instead of $E(k) \propto k^2$)? — Discuss this for both the 3D and for the 2D cases.

2. – ***Band picture of ELECTRONS in solids****: (Metals vs. Non-metals)*

Consider a 1D chain of N atoms with one atom per unit cell. Assume periodic boundary conditions and that each atom has Z valence electrons.

A) Show that you can fill exactly $Z/2$ bands with these electrons or, equivalently, that each band can accommodate $2N$ electrons.

B) On page 118, the pejorative term "spaghetti diagram" can be used to describe Figure 6.13. Still, it. Does show that silicon (Si) has four filled bands (for some values of $\mathbf{k}$, the energies of the bands are degenerate, but not for all). There are also four electrons per Si atom (not eight!). Explain why this is so.

C) Having an even number of electrons per unit is *necessary but not sufficient* for a solid to be an insulator/semiconductor. Give an example of an elemental solid that is a metal despite having an even number of electrons per unit cell.