Your Name:______________________________

Thinking about the "*mean free path*"

For starters, let's consider a moving *atom* (though, later on, for our experiment, we'll have to consider a moving *electron*) passing through a gas (for now, of atoms of the *same* species as our moving atom). Although the notation can cause some confusion, it is common to use the symbol λ to represent the "mean free path" between collisions, for the moving atom passing through the gas. This is *not* a wavelength! It's just that we run out of letters at some point, and so a certain amount of "recycling" must occur. Here, λ is just the average distance traveled by our moving atom, between collisions with the background gas atoms.

Special case (a): If the particular atom that we are tracking is moving *much* faster than the average speed of the atoms in the gas, then we may treat the background gas atoms as approximately stationary. This will help us to quickly *estimate* an approximate formula for the mean free path. It helps to draw pictures, whenever you can (!), so, ...

Draw, and clearly label, a figure containing an imaginary cylinder of volume $\pi a^2 \lambda$ where a is the "effective" ***diameter*** of an atom. At one end of the cylinder, draw, and clearly label, the center of our moving atom, which is moving along the cylinder's axis of symmetry. At the other end, perhaps a bit *off* of the axis of symmetry, draw the center of another atom that our moving atom will eventually collide with. Decorate the region outside of your imaginary cylinder by drawing in the centers of other atoms, but there must be *no other atoms* whose center lies within the cylinder. That said, it can be helpful to draw one or two atoms whose centers lie *just beyond* the sides of the cylinder.

Next, use dotted lines to draw circles around the center of each atom you've included in your picture. *Nota Bene*: a must be an atomic diameter and not a radius! Statistically speaking, this cylinder is a visual representation of the "free path" that our moving atom will take. Convincingly argue that this cylindrical volume will contain an average of *one* atom, and so the ***number density*** of the gas, n, must be given by $n = (1 \text{ atom})/\left(\pi a^2 \lambda\right)$.

Rearrange your equation to yield an initial estimate for the mean free path:

In general, we re-write this in terms of an effective cross-sectional area for scattering, σ:

$$\lambda = \frac{1}{\sigma n}$$

b) Were we to consider instead, *not* an especially fast atom, but an *average* one, then an additional factor of $\sqrt{2}$ would be introduced into the denominator. Also, using the ideal gas law, the number density of the gas is determined by the pressure and absolute temperature: $n = P/k_B T$. Including the factor of $\sqrt{2}$, go ahead and estimate a ***numerical value*** for the mean free path for nitrogen gas under standard conditions (1 atm = 101325 Pa, and 298 K). Be sure to keep track of your ***units***, explicitly!

c) Suppose now that, in preparation for thinking about the Franck-Hertz experiment, instead of considering a moving ***atom***, we move on to consider a moving ***electron*** passing through a gas (still consisting of the same *atoms* considered previously). Because they are so very light, electrons accelerated to Kinetic Energies of just 10 eV are already moving much faster than the mean velocity of (the much more massive) gas molecules at room temperature, and so we do not need that additional factor of $\sqrt{2}$ mentioned above, *but* we do need to make some other changes! Make a new version of your cylinder drawing and use it to ***argue*** that, given the relative size of the electron, it might make sense to reduce the effective scattering cross-section by a factor of four.

d) Estimate a ***numerical value*** for the mean free path of the electron moving through a neutral gas at a pressure of 10 Pa, which is as low a pressure as we can reach using the simple mechanical vacuum pump available in Modern Physics Lab. (No credit will be given unless you show your work.)

e) Clearly, even at a very low density of atoms (*i.e.*, low pressure), an electron is not expected to go long distances without suffering a collision, … but whether those collisions can be *inelastic* or must necessarily be *elastic* becomes a question about how much energy the electrons have, and whether or not that energy is sufficient to cause an atomic excitation.