**Exercise #4: Counting Statistics (redux)**

This experiment explores the *statistics* of counting experiments. Recall that, in performing their breakthrough work on radioactivity, Rutherford and his collaborators[1] developed methods for detecting, and counting, single particles emitted from radioactive substances. You know know that, today, we have a strong interest in the statistics of various kinds of *light* sources, and whether (or not) they are indeed random (and whether or not they exhibit "photon bunching"); not only are the following tests analogous to Rutherford's historic experiments with radioactivity, they prepare you for research in **Photonics**. *For this, read about the theory of the binomial, Poisson, Gaussian (normal), and Exponential distributions.*
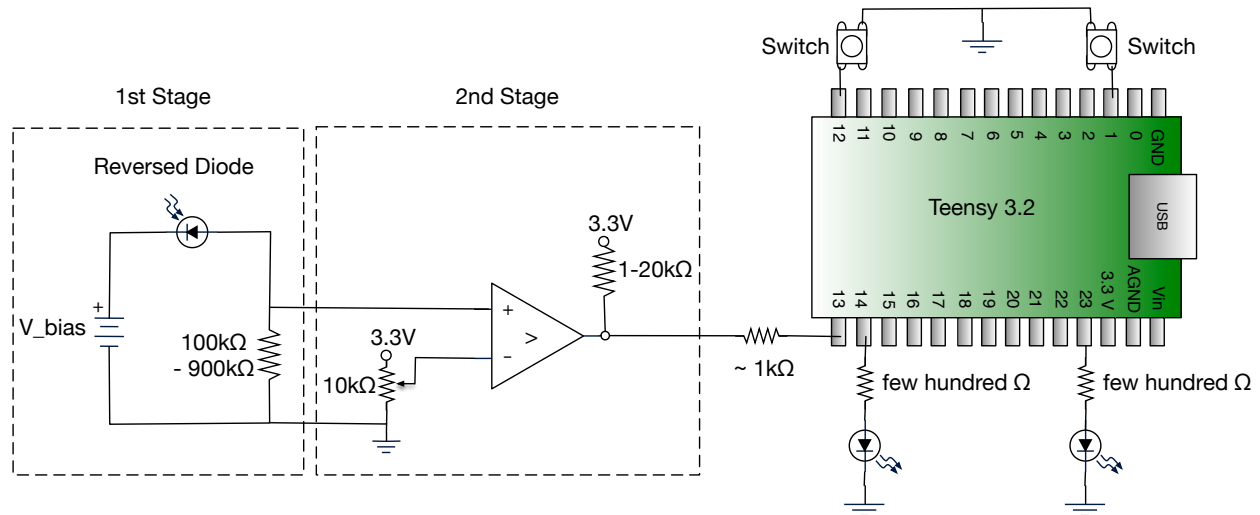
Procedure:

1. Although not all shown in the schematic above, all of the connections that you made last week, in the 2nd stage, should remain in place! [It is convention to not show "the rails" in schematics but, as you are now starting to realize, op-amp circuits will not work without them!] After ensuring that everything from last week is still working, **disconnect** the USB and **turn off** the V_bias applied to your SPAD.

2. Connect the output of the 2nd stage of your measurement chain (your discriminator) to the microcontroller's input: you do this by adding a 1K resistor that connects pin 7 of the comparator to pin 13 of the Teensy microcontroller. (You could have used a frequency counter to record data by hand, but the automated nature of a microcontroller data collection process makes things *much* easier.)

3. Connect a resistor (a few hundred ohms) to pin 14 of the Teensy, and then add, *in series*, a diode, which then connects to ground. This diode should be forward biased.
4. Repeat Step (c), with pin 23 of the Teensy. This diode should be forward biased.

5. Add a **button switch** that can toggle a connection between ground and pin 12 of the Teensy.
6. Add a **button switch** that can toggle a connection between ground and pin 1 of the Teensy.

7. Re-apply the V_bias applied to your SPAD, and re-connect the USB to the Teensy. Last week's signals should still be there.

A. If using an IWU computer, open the Arduino application. If using your own laptop, click the following link to prepare your system.

B. *Load*, via the "right arrow" icon at the upper left of the Arduino app, Henry's code: SPADCounter02.ino — *Skim* the code, noting that there are two modes of operation: you'll first use *FREQUENCY counting mode* instead of the *INTERVAL* counting mode, which will be used in Exercise #5. Note: an **integration time** must be specified. (This is your count interval, which you had often set to 30 seconds for the Geiger tube, but occasionally set to 10 min. Here, the default is 1000ms.)

C. In the Arduino application, click VERIFY. Ask for help, as needed. Once set, you can open the *Serial Monitor* (the upper right button on the Arduino app) to communicate with the Teensy.

D. Open an Excel spreadsheet and CLICK on a cell where you would like data entry to begin. Then , on the Teensy microcontroller, press the WHITE button, and *then* click on the particular **button switch** that can toggle a connection between ground and **pin 12** of the Teensy. DO NOT touch your computer until you **collect photon frequency data over *at least* 100 integration periods**. What environmental variables need to be kept constant for a valid dataset? Your frequency data points should all be between 1000-5000 (which may require you to *adjust* the V_bias applied to your SPAD and re-start your data run). To *stop* your data run, click the *button switch* that began your run. [If you have time, you might collect photon frequency data over *a couple thousand* integration periods: this won't change the standard deviation, but will yield a better approximation to the limiting distribution, improving the standard deviation *of the mean, $\sigma_m = \sigma/\sqrt{N}$*.] Import a valid dataset into *Igor Pro*. **Correct the data for the *background* signal**, if one exists. [Use Chauvenet's criteria, as appropriate, to remove data points.] Ensure thorough *documentation* of your methods.

Statistical Analysis:
For the entire *valid* dataset, calculate the mean $\bar{x}$, standard deviation σ, variance σ^2, and the standard deviation *of the mean*. An explanation of what σ_m means, and what it is used for, is *required*.

- Next, calculate the average absolute deviation $d' \equiv \langle |x - \bar{x}| \rangle$ for the whole *valid* set of readings. *If the data is Gaussian, you should be able to show (i.e., derive) that the average absolute deviation should tend toward*

$$d' = \sigma\sqrt{\frac{2}{\pi}}$$

  Calculate d' for your data set and see if it agrees with this expected relationship.

- Prepare a **histogram** of the data, complete with error bars. — As a "rule of thumb," choose your number of bins so that the highest bin has *something like* a fifth of your data. Both for dealing with histograms and for understanding counting experiments, please review:
  - Histograms and the Normal Distribution (Taylor 5.1-5.3, pp. 121 - 135)
    - The **required problem** on the *Exponential* Distribution (Taylor Problem # 5.6, p. 155)
  - The Poisson distribution (Taylor Ch. 11, pp. 245 - 254)
  - The Square-Root Rule for a Counting Experiment (pp. 48 - 49)
- You may compare your data to Gaussian or Poisson distributions, using the mean and σ from your data to create theoretical distributions of these forms. *Discuss the relative merits of each* of these for describing the collected data. A χ^2 test can provide an estimate of the goodness-of-fit between theory and experiment, but actually doing fits of your data to these functions is *extra*.
- Report the mean value of the frequency data and its associated uncertainty with the correct number of significant figures. Compare σ to the mean.