DIRECT CURRENT CIRCUITS I

[Written by Suzanne Amador Kane; Modified slightly for our purposes.]

Readings: Serway, Chapters 27–28.

At long last we are ready to begin work with Direct Current (DC) Circuits. DC circuits are those in which the voltages and currents are constant, or time-independent. Time-dependent circuits in which the current oscillates periodically are referred to as Alternating Current (AC) circuits. We will study AC circuits soon enough.

As you know, charge is the fundamental unit of electricity much as mass is the fundamental unit of gravitation. A significant difference is that mass is always positive (as far as we know), while charge can be either positive or negative. When charges flow, they form a current, just as when mass moves it has momentum. If we drop a mass, it "flows" down to a lower potential, and a potential difference in an electric circuit drives a current to the lower potential from the higher potential. Since charges come in both positive and negative flavors, electrical potential drives some charges up while others flow down. By analogy, if there were negative mass, it would fall up.

When a ball is dropped into a liquid, it falls more slowly due to the resistance of the liquid. Likewise, in a circuit we insert a "resistor" which impedes the flow of current. In a circuit, the effect is to lower the rate at which charge moves, effectively lowering the current which flows through the circuit. It is useful to note here that if we did not have a resistor, in effect it would be like dropping a huge object onto the Earth without air resistance – look at the craters on the Moon to see how undesirable this would be! In circuits, this is called a "short circuit" and it quickly drains the potential while also risking serious damage to the components (and the experimenter).

In an old fashioned mill, falling water was converted into mechanical power. Likewise, in a circuit the flowing charge does work as it moves from higher to lower potential. Since the work done in the mill depends on how much water falls through the gravitational potential, it is easy to understand why electrical power is given by $P = IV$, *i.e.*, the amount of current flowing through the potential difference.

DC Circuit Analysis

Ohm's Law

For many materials, the current that flows through the material is proportional to the applied voltage difference according to Ohm's law:

$$V = I\,R\,, \tag{1-1}$$

where R is a constant independent of V, but dependent on the material, geometry, and possibly other factors. Although Ohm's law is widely used in circuit analysis, it should be remembered that it is only an approximation that is seldom in practice satisfied *exactly*. Materials that obey Ohm's Law (have constant R) are referred to as Ohmic or linear devices, since the voltage is linearly proportional to the current. A resistor, an electronic device simply consisting of a length of wire or other conductive material connected to two leads, is the most common example of an Ohmic device.

Joule Heating

Consider a resistor R with a voltage V across it and a current I through it. As each element of charge $\mathrm{d}Q$ moves through the resistor from higher (+) to lower (-) potential, the amount of work done is $\mathrm{d}W = V\,\mathrm{d}Q$. Power is the rate of doing work: $P = \mathrm{d}W/\mathrm{d}t$. Thus we have

$$P = V\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right) = VI\,. \tag{1-2}$$

From Ohm's law, the power dissipated in a resistor is

$$P = I^2 R = \frac{V^2}{R} \tag{1-3}$$

The power dissipated by any component must be kept under 1/4 watt in this lab unless otherwise specified.

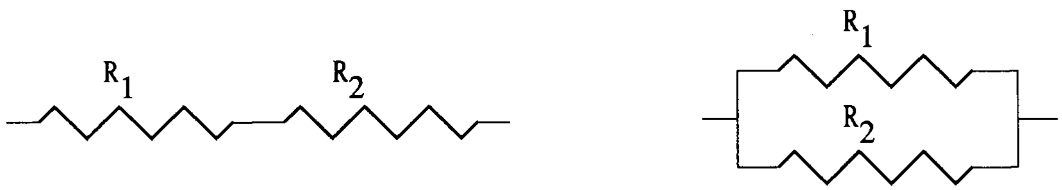
Note on units: when you consistently measure voltage in Volts, current in Amperes, resistance in Ohms, and power in Watts, no conversion factors are needed in any of the above equations. Be sure, though, to keep track of powers of ten implied in prefixes such as kilo and milli.

Prelab Problem 1: Calculate the maximum current that a 1/4 watt, 100 ohm resistor can handle.

Prelab Problem 2: If a 100-ohm resistor is connected across a 10 volt power supply, what power rating should the resistor have (at least)? Answer: At *least* 1 watt; 5 watts would be better. Note: If a 1/4 watt resistor were used, it would get VERY HOT and may eventually break apart!

Equivalent Resistance

In many circuits it is often possible to substitute a single resistance value for a complicated network of resistors. These networks occur unavoidably when you construct useful electronic circuits, as you will shortly see. A single resistor with "equivalent" resistance behaves the same as the original network. Two common methods of joining resistors are shown below.



Series Parallel

Figure 1. Series and Parallel resistor combinations.

Series: In a series combination, any current must flow first through one and then the other resistor. The equivalent resistance of two resistors in series is simply the sum of the two resistances:

$$R_s = R_1 + R_2. \tag{1-4}$$

Parallel: In a parallel combination, any current must flow either through one branch or the other, but not through both. The equivalent resistance of two resistors in parallel is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \tag{1-5}$$

Both of these rules can be extended to more than two resistors.

Kirchhoff's Laws

If a circuit cannot be reduced to a combination of series and parallel networks, it is necessary to use Kirchhoff's laws to analyze the circuit behavior.

Kirchhoff's First Law (junction theorem): The sum of all currents entering a junction is equal to the sum of all currents leaving the junction. This is equivalent to saying that charge cannot accumulate at a junction.

Kirchhoff's Second Law (loop theorem): The sum of the voltage differences around any closed loop in a circuit is zero.

Kirchhoff's Little-Known Third Law (spelling theorem): Kirchhoff is spelled with two h's and two f's.

Voltage Divider:

Consider the circuit in figure 2. The input voltage is said to be "divided" between the two resistors in series. The voltage across R_2 is given by

$$V_o = V\left(\frac{R_2}{R_1 + R_2}\right). \tag{1-6}$$

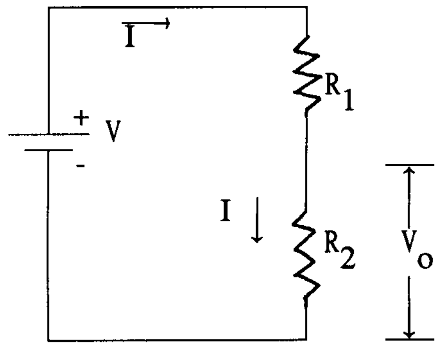


Figure 2. A voltage divider.

In some cases, V_o can then be used to drive a device that requires a voltage lower than V. Consider the following modification of the above circuit (figure 3): The resistor R_3 has been added in parallel to R_2. If $R_3 >> R_2$, then

$$V_3 = V\left(\frac{R_2}{R_1 + R_2}\right). \tag{1-7}$$

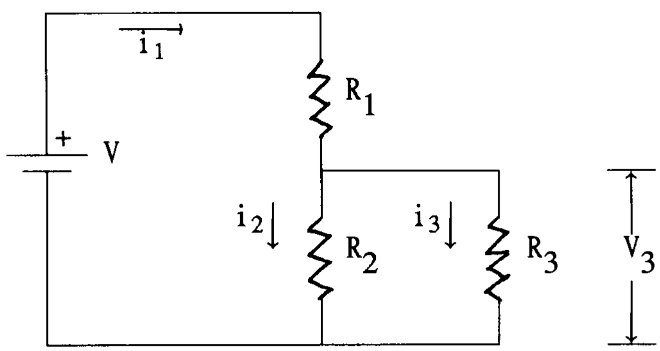


Figure 3. A voltage divider in use.

Prelab Problem 2-3. Derive equations (1-6) and (1-7).

Experiment 1-1: Measuring Resistance

You will be using a digital multimeter (DMM) to measure resistance and voltage. Spend a few moments and acquaint yourself with all the settings on the front panel.

WARNING: Meters, decade boxes, and resistors can be destroyed by excessive current. You must avoid mistakes by leaving one terminal of the power supply or battery disconnected until you are certain that the rest of the circuit is properly connected, and by checking with an instructor if you have doubts.

- Set the DMM so it will read resistance (*i.e.* push in the button labeled Ω). Touching the leads together (shorting them) should produce a zero reading. Note the format of the display when the leads are not touching, using the least sensitive scale of the DMM. This indicates a reading out of range, *i.e.*, an infinite resistance. An open circuit has infinite resistance. Some DMM's display this by flashing while others indicate it by using a 1 on the far left of the display. (Note: If the resistance of the object you are measuring exceeds the maximum the DMM's scale will show, it will register as out-of-range, and you must increase the range of the DMM by pressing a button corresponding to a higher resistance range to make an accurate reading.) For more information on DMM's, see Appendix D.

  Plug-in "breadboards" will be used to make connections in this lab. These kinds of devices are used by engineers to make prototypes of electronic instruments. *Make sure you fully understand what points are in electrical contact with each other before continuing*.

- Now select three different valued resistors in the range of 1 kΩ to 100 kΩ and measure their resistances individually. Do the readings agree with the color code and tolerance? The color code can be found in Appendix A. Use these resistors as R_1, R_2, and R_3 for the next parts of this lab.

  1) Measure the equivalent resistance for the series combinations of R_1 and R_2, R_1 and R_3, and R_2 and R_3. Use the breadboard to make these connections. Do these reading agree with the calculated value based on the individual measurements? Measure the equivalent resistance of all three resistors in series. Does this reading agree with the calculated value?
  Make sure you use your breadboard properly, as described in the pre-lab lecture.

  2) Perform the same measurements as in 1 for parallel combinations of the three resistors.

  3) Using your values for the three resistors, construct the circuit shown in figure 1-4 below. Measure the equivalent resistance of this network. Does the reading agree with the value you calculate?

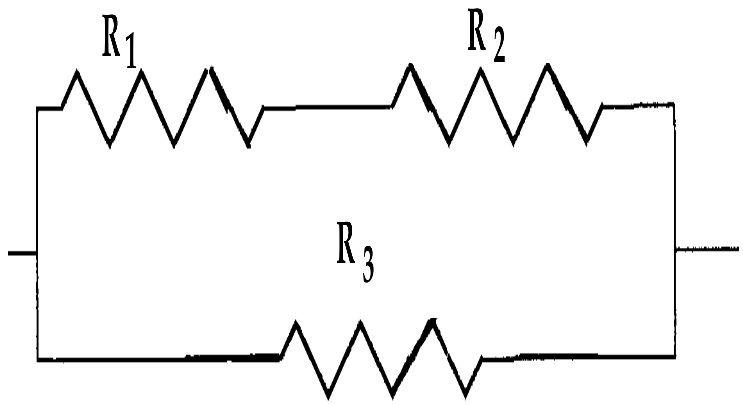


Figure 4: The resistor network for step 3 of experiment 1-1.

Experiment 1-2: An application of Ohmic conductors

While many metals and other common conducting materials have very linear (Ohmic) current vs. voltage (so-called I-V) curves, this case is really a special one for conductors in general. Even when Ohm's Law is followed, many materials have resistances that are functions of their environments. For example, special resistors can be made with resistances that vary with imposed strain, temperature, magnetic field and pH. Such devices are used as transducers (electronic devices which convert a physical quantity into a measurable signal.) As a common and useful case, we will work with a thermistor

Thermistors are used in a wide variety of applications where an accurate determination of temperature is needed. Once the thermistor is calibrated (the resistance vs. temperature plot determined), the thermistor can be used to provide a temperature measurement that can be directly used in an electronic control system.

1) Take a thermistor from the electronics bins and attach it to the DMM set to measure resistance. Determine its resistance at room temperature.
2) Place the thermistor into the nook of one lab partner's elbow (on the skin) and close that arm so that the thermistor is completely surrounded by and in good contact with the skin. Observe how the resistance changes and record the final resistance.
3) Assume that the temperature of the elbow is body temperature (37° C) and ask your instructor for the room temperature. Using this data and assuming the thermistor responds linearly (it does), plot the resistance vs. temperature response of the thermistor.

4) Predict, from your experience with the DMM and resistors, the minimum change in temperature that your DMM and thermistor could detect.

Experiment 1-3: Non-Ohmic conductors

The resistance of a light bulb is not well defined but varies with the applied voltage. That is, its I-V curve is not a straight line. This is an example of a non-Ohmic circuit element. To explore this phenomenon, obtain a graph of the current (i) through a bulb versus the voltage (V_L) across it, for voltages between 0 and 6V. Do not exceed 6V – you might burn out an element in your circuit. Take at least 10 points spaced out over the range 0–6V.

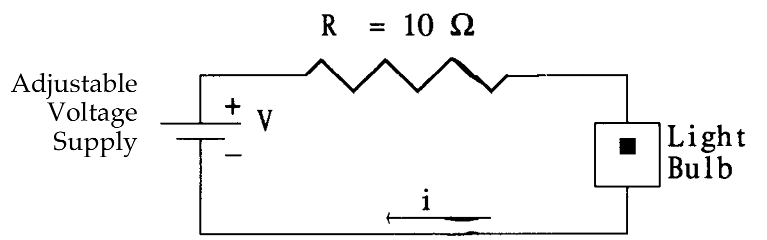


Figure 5. Circuit for measuring I vs. V for a light bulb.

1) To measure the current i, construct the circuit in figure 5 using a series resistance R = 10Ω. The power resistors in the bins should be used for this purpose because of their high power ratings. (Remember that even 6V across 10Ω gives 0.6A, or P=VI=3.6W, far in excess of the 1/4 W rating of the most common resistors: you may need to use *combinations* of resistors to keep from exceeding the rating of any one resistor.) By assuming Ohm's Law for the resistor, you can obtain the current i through the bulb from the measured values of V_R and R.
2) Using KaleidaGraph, plot V_L vs. i. (It is possible to define the light bulb's resistance for a particular combination of current and voltage, but you will find that the resistance is ***not*** constant over the full voltage range: the slope between the lowest points should differ noticeably from the slope between the highest points.)
3) Discuss the behavior of the bulb's resistance as a function of applied voltage. Can you give any reason (based on the behavior of the filament in the bulb on a microscopic scale) for the variation in resistance? Since you will probably not have discussed this in any of your courses, just give it your best shot and feel free to extrapolate wildly from what you know from either physics, chemistry, or biology
4) Save your lightbulb circuit for the next experiment.

Experiment 1-4: Voltage Divider

A potentiometer is a type of variable resistor based on the idea of a voltage divider. A potentiometer (commonly called a "pot") consists of a resistor with an additional third sliding tap.

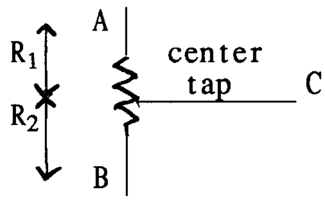


Figure 6. Inside a "pot".

The resistance between points *A* and *C* (R_1) depends on the position of the center tap. The resistance between *A* and *B* ($R_1 + R_2$) is always constant. If *A* and *B* are attached to a voltage supply (V_S), the output voltage (V_{BC}) can be varied continuously from 0 to V_S by sliding the tap.

1) Observe the change between pairs of terminals with your DMM. Which is connected to the sliding tap?
2) Set your voltage supply to 6V and attach the fixed terminals of a 1kΩ potentiometer to it. Observe the voltage of the third terminal as you slide the tap.
3) The voltage supplied to a light bulb can be varied using a potentiometer, hence the brightness of the light can also be varied from its full intensity to zero. Draw a design for how you would modify your light bulb circuit from the last exercise to install a dimmer (based on a potentiometer) and try it out.

You could construct a room light dimmer this way, however much of the voltage drop, and thus the power, would go into heating a resistor rather than producing light. In reality, light dimmers work by quickly turning on and off the current to the bulb so that all the power does go to the bulb, but not continuously. In a slightly more complex example, the volume control knobs on a stereo also control the resistance of a potentiometer that forms a voltage divider. In this case, the voltage that is being varied is an audio signal read from a CD or tape and converted into a voltage, controlled by the volume pot, which can drive the speakers.

Experiment 1-5: Designing a Voltage Supply

As a final exercise, try a small, practical electronic design project. Frequently, one is faced with a need for many different voltages in a circuit, but your power supply provides only some fixed standard voltage output. For example, a particular Radio Shack motor runs at 6V, but common supplies output +5V or +/-15V. You need to provide the voltage yourself using a circuit built for the purpose. Using your breadboard and the electronics you are supplied with, design a voltage supply that provides the following configuration of voltages across its output terminals:

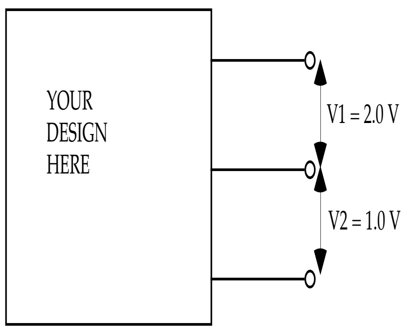


Figure 7

The power source inside your home-built voltage supply will be 1.25 Volt batteries (C cells), available with holders from your instructor. The batteries can supply only ~10mA of current, so plan the resistors in your design taking the total current flow into account. You will also want to limit your resistor choice to resistors under 20kΩ, to make your life easier at the start of DC II next week. Make a drawing of your design, explain why it should work, and describe how you tested it out.

Make sure you have a good circuit diagram in your write-up.