Mechanical Chaotic Oscillator

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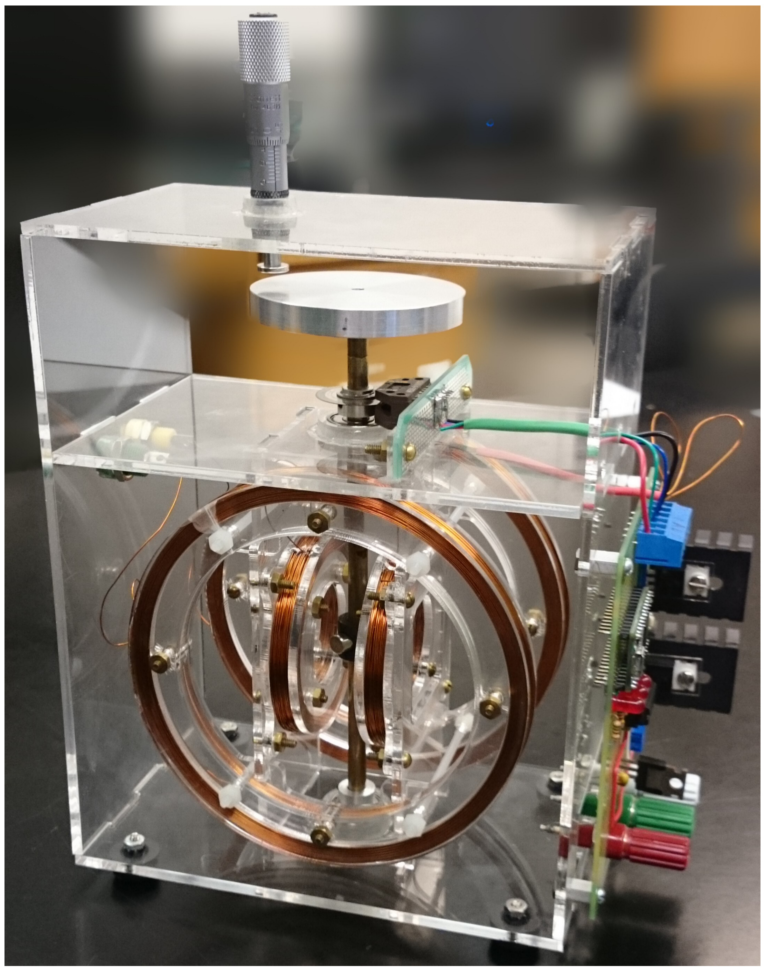
Abstract

This project objective is to create a tool to analyze the behavior of a chaotic system in the advanced lab environment. Here we explain the design and modeling of a chaotic oscillator along with fabrication and assembly methods. Processes for obtaining and processing data, as well as electrical design are discussed. Using a USB and computer, the user is given control over system parameters and access to system diagnostics. The resulting apparatus is a nonlinear oscillator capable of producing a chaotic trajectory while providing a unique hands-on way to study chaos.



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1 Introduction

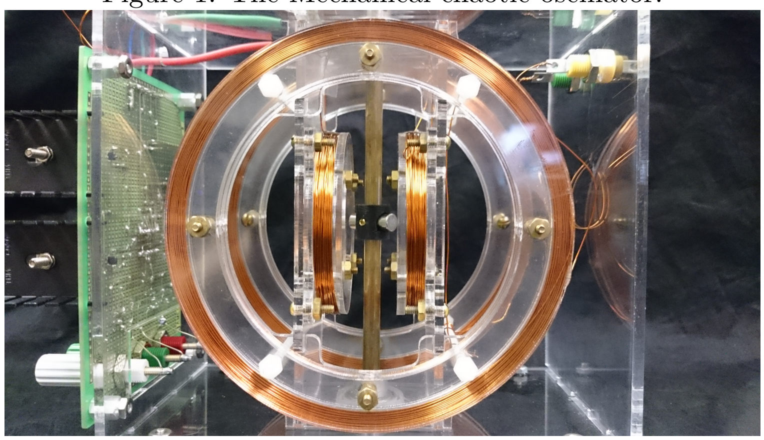
For most undergraduate physics students, chaos theory is a concept of very little if any exposure. Perhaps a student may get an opportunity to study chaos in a computational physics course where numerical methods provide a manageable treatment of nonlinear systems. Nonlinear dynamics and chaos are prominent features in nature and deserve a thorough introduction in the undergraduate physics curriculum. The Advanced Lab is an excellent environment for undergraduate students to explore chaotic systems in detail. The ability to observe a nonlinear system approach chaos gradually through a succession of period doubling bifurcations would make this introduction more intuitive. This would also give a student a better understanding of what it means for a system to be deterministic versus random. This paper explains the process of building a mechanical chaotic oscillator that can serve this purpose.

It is not difficult to build a system that exhibits some form of chaos, a simple double pendulum would do the trick! However, it is no trivial matter to create an apparatus that does so while allowing for complete user control over system parameters and precise measurement of the state of the system as time evolves. These are some of the requirements needed for this project to be successful in the Advanced Lab setting. The mechanical chaotic oscillator or MCO presented fulfills these needs in an effective and elegant way.

2 Concept of Design

The Helmholtz coil has always been a popular lab technique for creating a uniform magnetic field for experimentation. It also has the quality that the strength of the field it produces is linearly proportional to the current through its coils. This provides the convenience of precise field control, a feature we make significant use of with the MCO.

Figure 1: The Mechanical chaotic oscillator.



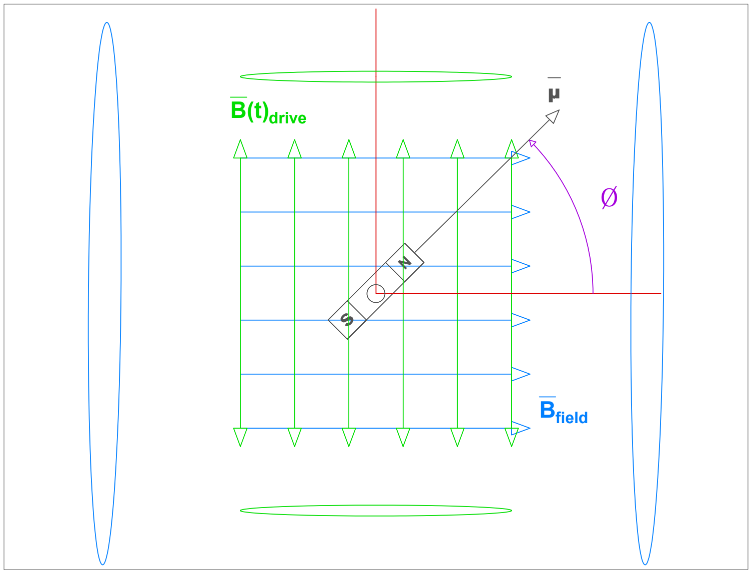
The apparatus is one involving magnetic interactions so nonmagnetic materials were used wherever possible to minimize interference. The apparatus uses two Helmholtz coil pairs, one to provide a constant magnetic field within a given region of space and the other to give a perpendicular time varying field component in that same region. This region contains a magnetic dipole connected to a rotating shaft that supports an aluminum inertial disk. The inertial disc supplies both rotational inertia and a means for magnetic damping. An optical encoder with a quadrature output signal tracks the position of the dipole. This signal is read by a microcontroller programmed to retrieve and process this information, as well as generate the signal for the time varying field. The microcontroller is able to receive specific commands through a serial connection. This lets the user control system parameters, obtain system diagnostics and access a help menu to learn how to use the apparatus from a computer. The position and time data for the system is sent from the microcontroller to a computer for further processing and use. This apparatus is a damped non-linear oscillator that allows the user control of the system through a serial connection.

3 Modeling the System

The equation of motion was determined in the variable ϕ and using numerical methods a phase space diagram was produced along with a strange attractor. The theoretical model provided a magnificent Poincaré plot showing this strange attractor. This occurred after adjusting system parameters such as drive frequency ω_d, damping coefficient β, magnetic moment μ and field strengths B_{field} and B_{drive}. Equation (1) below is the equation of motion we obtained for the MCO system where I is the moment of inertia determined by the mass distribution of the inertial disc and shaft.

$$\ddot{\phi} = \frac{\mu}{I}[B_{field} \sin\phi + B_{drive} \cos\phi \sin\omega_d t] - \frac{\beta\dot{\phi}}{I} \tag{1}$$

Figure 2: Diagram of magnetic dipole in uniform field region.



This second-order nonlinear differential equation was then written as a system of two first order equations, (2) and (3). Here we denote the rotational frequency of the dipole as ω_ϕ to differentiate from the drive frequency ω_d. These equations where then plotted in phase space using a Python program.

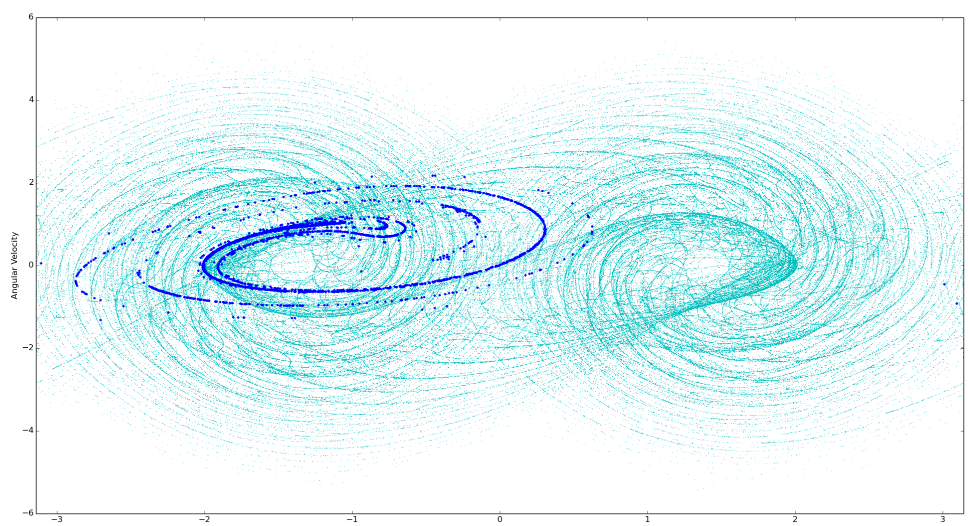
$$\dot{\omega}_\phi = \frac{\mu}{I}[B_{field}\sin\phi + B_{drive}\cos\phi\sin\omega_d t] - \frac{\beta\omega_\phi}{I} \tag{2}$$

and

$$\dot{\phi} = \omega_\phi \tag{3}$$

The figure below shows the resulting phase space plot of our theoretical model for 4000 laps of the drive frequency. The dark blue set of points are obtained by plotting one dot at the same point in each period; they show the strange attractor for this chaotic motion.

Figure 3: Chaotic Oscillator phase space plot with strange attractor.



4 Materials and Fabrication

The MCO is made up of many components and it is important to choose the right materials for each. The system is magnetic and so magnetic materials would effect the resulting trajectory if they where in close proximity to the dipole. While in some cases magnetic materials were used, the majority of the apparatus is nonmagnetic and no magnetic materials are present on or inside the coils other than the rotating dipole.

Case and Coils

The majority of the apparatus is made of acrylic. We were able to design these parts in AutoCAD and cut them from acrylic sheets using a laser cutter. This includes the case which uses a tongue and grove technique for assembly (bonded together using solvent welding) and the coils which are each comprised of four layers of acrylic held together with brass fasteners. Once assembled, the coils were wound on a lathe using 26 gauge magnet wire with 170 turns each. The two sets of coils are secured together using coil mount plates with brass and nylon fasteners.

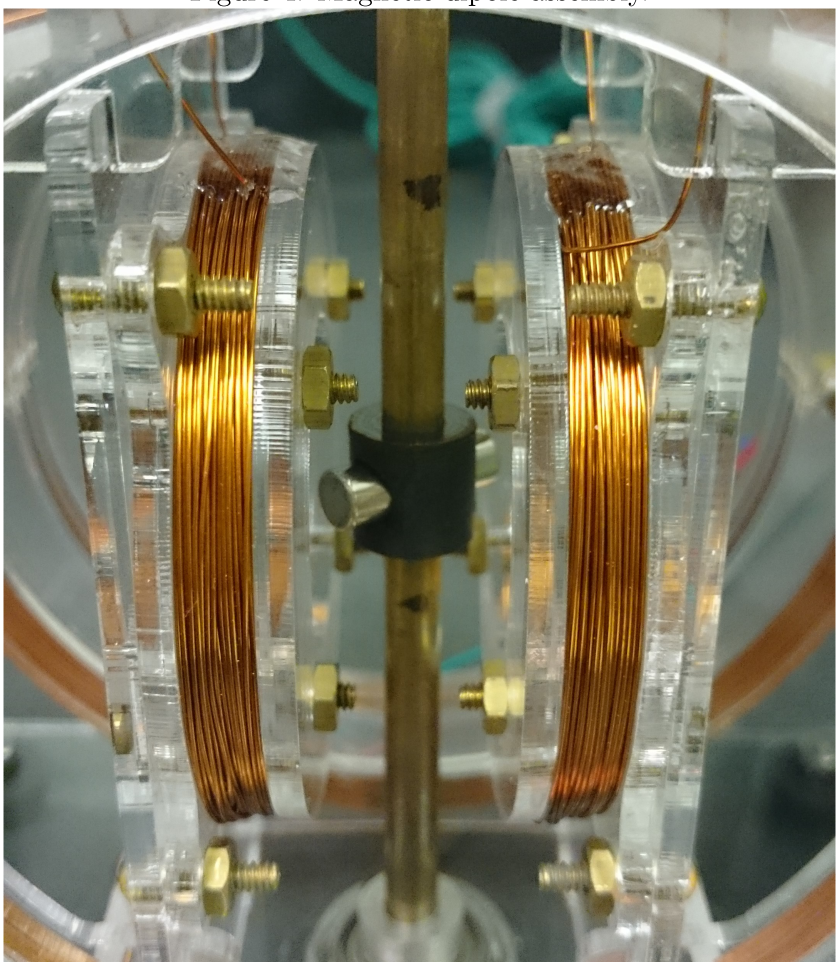
The Shaft and Its Components

For the shaft, again we needed a nonmagnetic material and so we used brass. The shaft serves three main purposes, support the inertial disk, hold the optical encoder disk in place and hold the magnetic dipole in place, while rotating freely. The shaft was chosen to be quarter inch in diameter because the bearings, encoder disk and inertial disk all had a quarter inch drive.

Bearings and friction

The bearings that hold the shaft in place are mounted in two locations on the case. One bearing is set in the base of the case centered below the coils and the other is set in the shelf just above the coils. These are made of steel and could have some small effects on the dipole's trajectory; the effect should be minimal though due to the radial symmetry. It is worth mentioning here that the rotational axis is vertical, thus eliminating gravitational effects. The only uncontrolled torque on the rotor is from the bearings. We assume that these small frictional contributions are negligible compared to the magnetic damping.

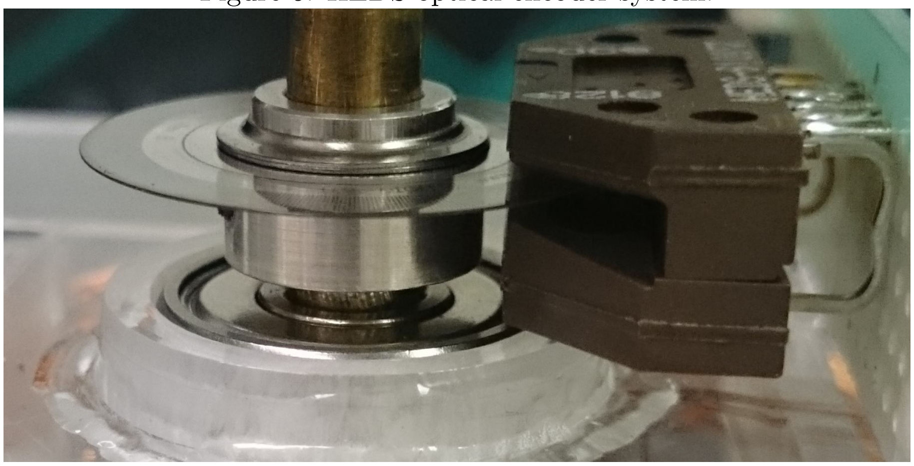
Figure 4: Magnetic dipole assembly.



The magnetic dipole

The magnetic dipole is mounted on the shaft at the center of the uniform field region between the coils. The dipole is made of two cylindrical magnets with a depth, diameter and spacing of a quarter inch. The mount that holds the magnets in place was designed in AutoCAD and printed in ABS on a 3D printer. The mount slides onto the shaft into position and held in place with a brass set screw.

Figure 5: HEDS optical encoder system.



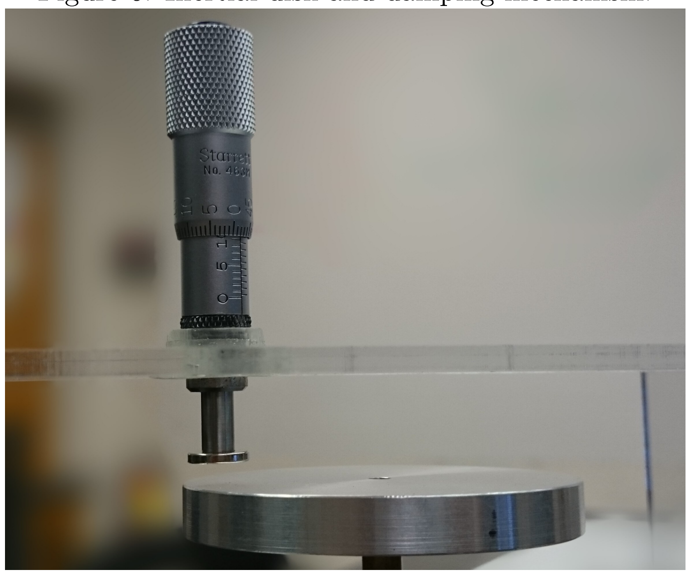
Optical encoder disk and module

Located just above the shelf at the top of the coils is an encoder disc that is used to track the position of the shaft. The disc is paired up with a HEDS optical encoder module that is mounted next to the shaft on the upper shelf face. The HEDS module gives a quadrature output signal that is processed by a microcontroller.

Inertial disk and damping

Above the optical encoder at the top of the shaft is an aluminum inertial disk. As the name suggests the inertial disk provides the majority of our mass. We were able to use it as a means of magnetic damping as well. This was achieved by mounting a micrometer screw adjuster in the top face of the case with a magnet attached to the extending end. The micrometer screw allows for a precise and measurable approach to the aluminum disk. When the disk rotates in proximity to the magnet the changing magnetic flux through the disc will create eddy currents within the disk and according to Lenz's law a drag will occur on the disk opposing the motion.

Figure 6: Inertial disk and damping mechanism.



5 Electronics

The MCO has an on-board microcontroller that is programmed to perform three main processes: produce a user controlled drive signal to the circuitry which supplies a current to the drive coils, process the quadrature signal from the optical encoder to get the trajectory for the dipole and provide an interface between the MCO and the computer.

Drive coil circuit

The output signal from our microcontroller's DAC is an analog sine wave varying between 0 and 3.3V. We however would like to produce an oscillating current through the coils capable of reaching 1 to 2 amps. This requires some circuitry and external power. The method used goes as follows.

First we shifted and amplified the DAC signal using a summing amplifier. The amplifier has two inputs, one from the DAC signal at 0-3.3V and the other connected to a -5V regulator through a trimpot and buffer circuit. The trimpot allows for manual zeroing of the summed signal. The signal is then amplified to a voltage of $\pm$ 10V by putting the appropriate valued resister on the feedback loop of the op-amp.

Next we use a push-pull circuit to drive the coils. The now 10V peak

Figure 7: Electrical schematic for the MCO.

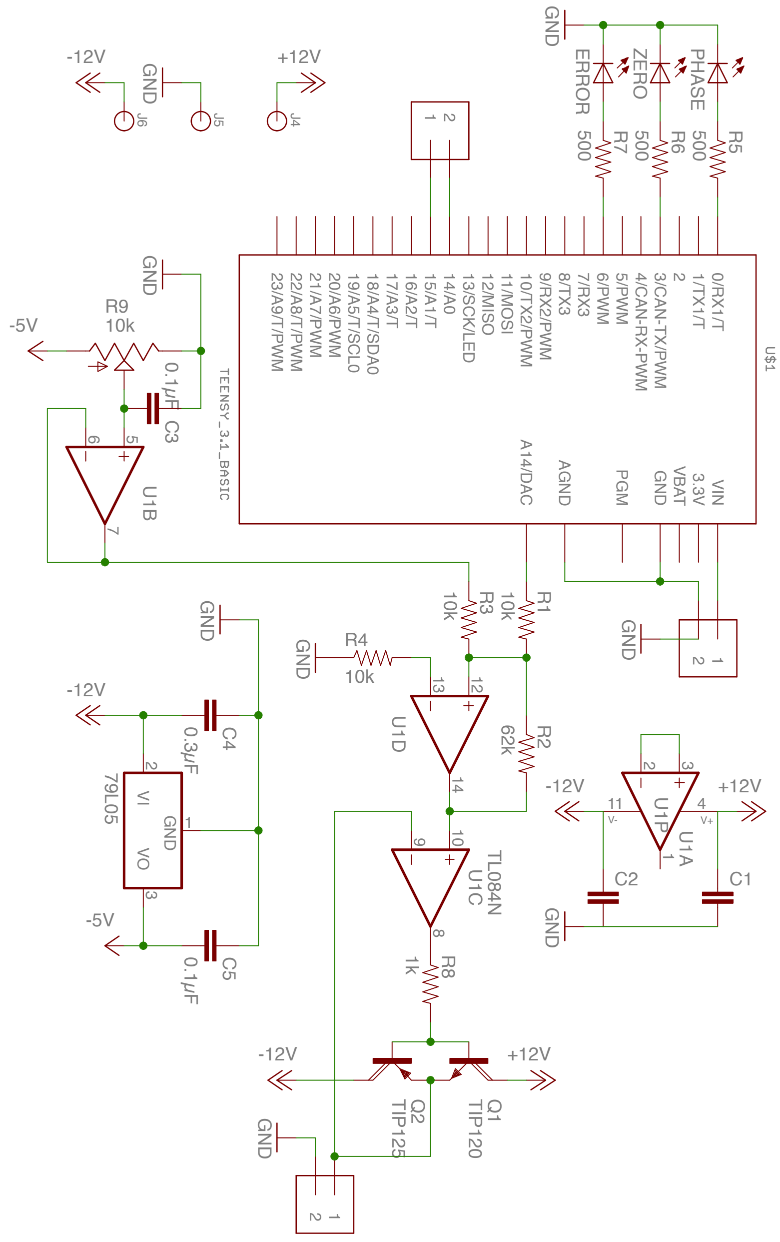
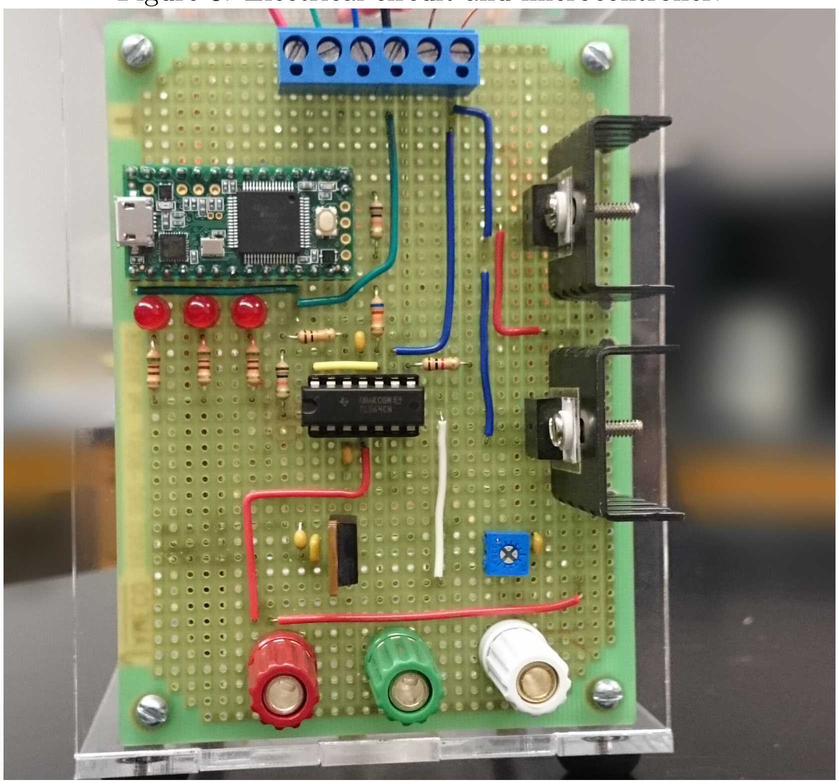


Figure 8: Electrical circuit and microcontroller.



AC signal is fed to the positive input of an op-amp. The output is sent to the base of two transistors simultaneously. The MCO circuit uses the TIP 120(NPN) and TIP 125(PNP) power transistors supplied with +12V and -12V respectively. Both emitters are connected together and out to the coils. The coil lead is also connected to the negative input on the op-amp, so as the input voltage varies between +10V and -10V the power transistors are letting through a smoothly varying alternating current for supplying the coils. Even though this process is not normally linear for transistors, in this case the feedback from the op-amp forces a linear response.

Data Processing and Serial Communication

The microcontroller is tasked with resolving the quadrature signal from the optical encoder module into position data. It also tracks time, so velocity data can be obtained. The MCO program developed by Dr. Ayars uses one of the quadrature lines to trigger an interrupt to track the motion of the dipole and the other one to specify the rotational direction at the time of the interrupt.

The microcontroller can report this time, drive phase and position data to the computer in real time using the IEEE 488.2 serial command protocol. The microcontroller also allows for the user to submit commands to and request information from the MCO in real time. This is done through the serial protocol, and can be used with many software options such as LabVIEW. The table below lists the specialized commands created for the MCO.

Table 1: List of commands for the mechanical chaotic oscillator.

Command	Explanation
*IDN?	Equipment and firmware ID
∗RST	Reset to default condition
∗TST	Test for centering adjustment (not yet implemented)
∗ESR	Report (and clear) error status register
FREQ (?) or (value)	Request (?) or set (value) drive frequency
TRAK (?) or (1/0)	Request tracking status (?) or set tracking on (1) or off (0)
REPT (?) or (1/0)	Request reporting status (?) or set reporting on (1) or off (0)
COIL (?) or (1/0)	Request coil-drive status (?) or set coil on (1) or off (0)
AMPL (?) or (value)	Request (?) or set (value) amplitude, 0-1000
ZERO	Define current position as zero
POSN?	Request current position
TIME?	Request current time in microseconds
SAVE (0-9)	Save current parameters in slot (0-9)
LOAD (0-9)	Load saved parameters from slot (0-9)

6 Results

The resulting apparatus is a nonlinear oscillator capable of producing a chaotic trajectory while providing a way of adjusting all parameters. Controlling the system through a serial connection and Labview makes it more useful. The basic commands and easy LabView interfacing allow students to spend more time learning with less confusion. Some future modifications planned for the MCO are creating a Labview GUI to control the system, using bearings with less friction and a higher resolution encoder for tracking the trajectory with greater precision.

7 Conclusion

The Mechanical Chaotic Oscillator offers the student a way to learn about nonlinear dynamics while having substantial control of the system. The apparatus provides the student with the tools for acquiring precise data while observing the process with their own eyes.