***Butikov* HW Set #3** **Exercises 2.1 and 2.4 from Chapter 1**

(Moving on to) Case 2: DAMPED Free Oscillations:

2.1 – The Sequence of Maximal Deflections:
Hint: note that this question (along with all the rest in this homework set) considers the case where damping is linearly proportional to the angular velocity. On the top right of the main simulation panel, be sure that you do now (finally) use the *checkmark* for "Viscous friction."

Re-read the first part of Sect. 1.3 from Butikov Chapter 1 (just *a couple of paragraphs*, ending with the sentence that reads "That is, the ratio of successive terms in this infinite geometry progression is less than unity by the small value …") — As described there, under the action of weak viscous friction, the sequence of maximal deflections of a free, damped linear oscillator forms a decreasing *geometric progression*: each consecutive maximal deflection is smaller than the preceding one by the same factor, $\exp\left(-\gamma T_0\right) \approx 1 - \gamma T_0$. Here, we've used a **Taylor Expansion**, which is why what follows is true only in the limit of *weak* viscous friction.

Recall that $T_0 = 2\pi/\omega_0$ is the "natural period" that would be found in the limit of no damping. Butikov, on page 4, shows that for the case of relatively weak damping, the **fractional difference** between the observed frequency of oscillation and the ideal frequency expected in the limit of damping, is proportional to the *square* of the small parameter γ/ω_0. In other words, there really isn't much change in the period from what you'd calculate using the ideal model with no damping.

In many engineering applications, the strength of the damping is commonly characterized by a dimensionless parameter called the *quality factor*, which is defined by:

$$Q \equiv \frac{\omega_0}{2\gamma}$$

We might loosely describe Q as the (rate of return)/(rate of loss).

(a) **Calculate** (showing your steps, and using enough words to make your steps clear) a predicted value of the quality factor Q at which the amplitude **halves** during every two complete oscillations.

(b) Use your value calculated in part (a) as input into in your computer **experiment**, and *verify* the theoretically predicted *constant* ratio of successive maximal deflections. Confirm that this ratio does not depend on the initial conditions (though picking even numbers for the initial amplitude may make it easier to see, more closely, whether the amplitude has indeed been halved after two cycles, as it would then coincide with a **tick mark** on the graphs supplied). Remember, Butikov call this is a "virtual lab." That's why all of your Butikov homework has been going into your lab notebook! — Here, you need to use all of the normal habits expected or a lab notebook including, of course, **showing** whatever you observe in an experimental test (including screen clippings), writing down a guide for the reader, and writing down your thoughts about it!

(c) This part of the question is not about the decay in amplitude, but about the shift in **period** as damping is added. See p. 4 of Butikov Ch 1, Eqn. 4: the period is predicted to change as damping is added, and you *certainly* can no longer say that damping is negligible if the amplitude halves every two cycles: For the value of the quality factor at which the amplitude **halves** during every two complete oscillations, evaluate the *increment* (in percent) of the *predicted* period of oscillations, including damping, with respect to the "natural period," T_0. (Recall that T_0 is the period that would be predicted from the ideal model with no damping.) Can you ***detect*** the increment in the simulation experiment? The marks on the time axis correspond to integer numbers of "natural periods," T_0 (which, again, is what you'd predict *without* damping).

2.4 – The Phase Trajectory of Damped Oscillations:

$Q = 0.5$ corresponds to a system that is "critically damped."

For $Q < 0.5$, the system is "overdamped" and oscillations do not occur.

For $Q > 0.5$, the system is "underdamped," and the phase trajectory of the oscillations that are associated with free decay will be a **spiral** that makes an infinite number of gradually shrinking loops around the focus, located at the origin of the phase plane. This focus corresponds to the state of rest in the equilibrium position, and the phase trajectory approaches it asymptotically.

(a) In the underdamped limit, how, *specifically*, does the radius of these loops change while the curve approaches the focus? Can you provide an **equation** that applies as we vary *Q throughout* the underdamped regime?

(b) Does the time interval during which the representative point makes one revolution of the spiral ***change*** as the loops of the curve shrink?