Butikov HW Set #1 Exercises 1.1 - 1.4 from Chapter 1

Case 1: UNDAMPED Free Oscillations:

Butikov calls this is a "virtual lab." That's why all of your Butikov homework should go into your lab notebook! — Here, you need to use all of the normal habits expected or a lab notebook including, of course, whatever you observe in an simulated experiment *test*, write it down, and think about it!

1.1 – The Initial Conditions and the Shape of the Plots:

By "**Initial conditions**" we mean specifying both:

- the initial angular **position**, φ_0, and
- the initial angular velocity, Ω (or the initial angular **momentum**, $J\Omega$, which amounts, to within a constant, to stating the initial velocity).

In the absence of friction a linear oscillator executes simple harmonic motion, which is characterized by purely harmonic time dependence of the angular displacement and of the angular velocity.

(a) i) What initial conditions give rise to oscillations of **cosine** time dependence?

ii) What initial conditions give rise to oscillations of **sine** time dependence?

iii) Suppose that you want to get oscillations with an angular amplitude of 90°. If we are given that the torsional pendulum is released "from rest," that is, such that the initial angular velocity, $\dot{\varphi}(t = 0)$, is zero, then what initial angular displacement, $\varphi_0 \equiv \varphi(t = 0)$, will ensure the desired angular amplitude of 90°?

(b) In order to obtain that same desired amplitude of 90°, what initial angular velocity, $\Omega \equiv \dot{\varphi}(t = 0)$, *ought* you impart to the oscillator, if it starts out at rest in the equilibrium position? Remember, that the initial angular velocity Ω must be expressed, for input into a *simulation*, in units of the natural frequency ω_0. Verify your answer by performing a computer "experiment," using the appropriate initial conditions.

Hint: note that this question (along with all the rest in this homework set) considers the special (idealized) case of NO FRICTION. In that case, p. 3 of Butikov Chapter 1 provides the general solution in Eqn. (3).

To confirm that this all works "as advertised," you need to download the executable jar file (Java archive) OscillationsE.jar in which several simulations are packaged, for which you must read the online description. When you start the Java applet, the list of available simulations appears. Select the simulation *Free oscillations of the Torsion Spring Pendulum* from this list. [Rather than using the sliders to input values, you can type them in directly. For this problem, there should be no checkmark for "Viscous friction."]

By the way, this simulation can also be used to confirm our answers to (a) part (i) and part (ii) by checking the list of predefined examples in the simulation (using the check-box under the image of the torsional oscillator), and clicking (on the left side) to show the graph of angle *vs*. time.

1.2 – Maximal Deflection and Conservation of Energy:

In the absence of friction, imagine exciting an oscillator, that starts out initially at rest **in the equilibrium position**, by a push that produces an **initial angular velocity** $\Omega = 2\omega_0$.

(a) Calculate the resulting angle of maximal deflection, φ_{max}, using the law of the conservation of energy.

(b) Verify your result via computer experiment. Note that the simulation program performs the numerical integration of the differential equation independently of conservation laws, such as the conservation of energy. That is, those laws were not used in the program, so it is not a "circular argument" to use the results of the computer experiment as a test. Note, also, that you should have simplified your result to part (a) to yield a numerical result. [Watch your units!] Otherwise, it would be ***impossible*** for you to use the numerical simulation to confirm your result!

1.3 – The Phase Trajectory and the Initial Conditions:

Compare the motion of the **representative point** along the phase trajectory of a ***conservative*** oscillator (meaning, again, **in the absence of friction**) with the time-dependent plots of the angle of deflection and of the angular velocity.

a) How is the phase trajectory changed if you change the initial conditions?

b) Does the direction of the motion of the representative point along the phase trajectory depend on the initial conditions?

c) Is it possible that phase trajectories for different initial conditions coincide? If so, formulate the requirements for the coincidence.

Phase space = Position & Momentum, as introduced at the start of Butikov Section 1.4.
In the simulation, make sure that you have checkmarks on the left, to show both "Graphs *vs*. time" and "Phase diagram." You might want to reset the timer, and then, instead of clicking on START/STOP, try making use of the STEP button. On the "Graphs *vs*. time," the black curve shows the angular position, and the red curve shows the angular velocity. The "Phase diagram" is a plot of angular position *vs*. angular velocity (which is proportional to angular momentum, and given that we tend to use scaled dimensionless parameters, the same kind of information is being displayed as if we had made a graph of position *vs*. momentum).

1.4 – Elliptical and Circular Shape of the Phase Trajectory:

a) Prove analytically that the phase trajectory of a conservative linear oscillator is an ellipse with its center at the origin of the phase plane. Use the general solution of Eq. (2), expressed by Eq. (3). What are the semiaxes of the ellipse?

b) Show that the elliptical shape of the phase diagram of a conservative linear oscillator follows immediately from the law of the conservation of the energy.

c) What scale on the axis of the ordinate (the angular velocity axis) of the phase plane produces a circular phase trajectory?

d) Does the time interval during which the representative point passes along one loop of the phase trajectory depend on the initial conditions?