Name: ________________________

Partners:______________________

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Date: ________________________

Deflection of an Electron in a Magnetic Field

Purpose

In this lab, we use a Cathode Ray Tube (CRT) to measure the effects of an electric and magnetic field on the motion of a charged particle, in this case the electron.

Theory

The Lorentz force law, Equation 1, tells us that a charged particle experiences a force in an area where there exists an electric or magnetic field.

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{1}$$

where F is the force exerted on the charged particle, q is the charge of the particle, E is the electric field, v is the velocity of the charged particle and B is the magnetic field. When an electron ($q = -e$), is in a magnetic field, where $E = 0$, the electron experiences a force given by Equation 2.

$$\vec{F} = -e\left(\vec{v} \times \vec{B}\right) \tag{2}$$

To examine the motion of an electron in a magnetic field, we will use a cathode ray tube. To examine these effects, we use a CRT. Enclosed within the cathode ray tube is an electron gun, Figure 1, which will be used to produce electrons with given energy.

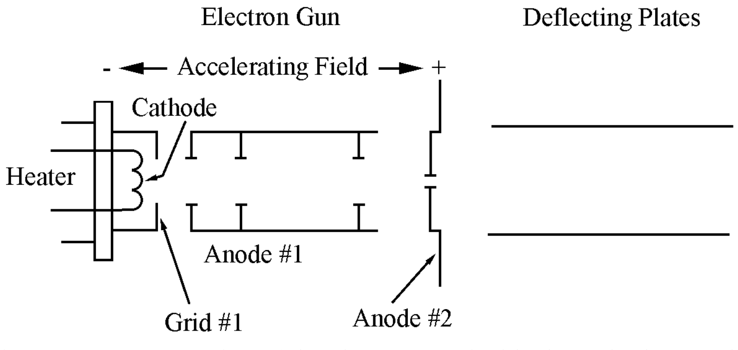

Figure 1: A cross-section of the electron gun inside the cathode ray tube.

The electron gun will emit electrons with a kinetic energy equal to the charge of an electron times the accelerating voltage, V_{acc}. These electrons will be deflected by a magnetic field that is generated by a current-carrying wire. In this setup, the electron is moving perpendicular to the magnetic field. Thus, the force on the electron is:

$$F = evB \tag{3}$$

Additionally, we know from last semester that, when the force acting on a body is perpendicular to the motion, the resulting motion is circular. Using this and conservation of energy ***SHOW*** that the electrons will move in a circle of radius *R*, given in equation 4:

$$R = \frac{m}{eB}\sqrt{\frac{2eV_{acc}}{m}} \tag{4}$$

The radius of curvature is related to the spot that is made on the screen, as seen in Figure 2. The electron beam is traveling from the left (where it is emitted by the electron gun) to the anode, and then on towards the screen. In the absence of a magnetic field this trajectory would be a straight line path, a distance *S* long; however, when a magnetic field is added (into the page) the electron beam will curve away from its original course, as seen in Figure 1.

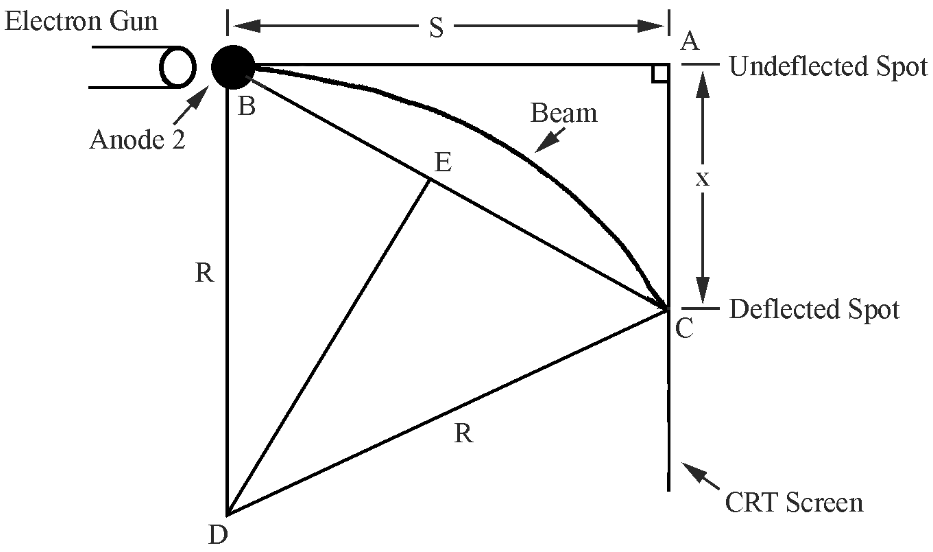

Figure 2

If the magnetic field is uniform, the force will be constant and the radius of curvature, *R*, is fixed. We can use geometry to determine how *R* is related to measurable quantities.

1. Δ BAC is similar to Δ DEB, since 2 sides are mutually perpendicular.

2. Using the Pythagorean theorem:

$$(\overline{BC})^2 = \mathrm{S}^2 + \mathrm{x}^2$$

3. For similar triangles, the ratio of sides is equal. Also, $\overline{DE}$ bisects $\overline{BC}$. Thus:

$$\frac{x}{\left(\overline{BC}\right)} = \frac{\left(\overline{BE}\right)}{R} = \frac{\frac{1}{2}\left(\overline{BC}\right)}{R}$$

4. From this, ***<u>you</u>*** should be able to ***<u>SHOW</u>***:

$$2Rx = \left(\overline{BC}\right)^2$$

5. Using our result from line 2 and some algebra, ***<u>you</u>*** should be able to ***<u>SHOW</u>***:

$$R = \frac{S^2 + x^2}{2x} \approx \frac{S^2}{2x} \qquad (5)$$

where *x* is the deflection of the electron beam and ***S* = 0.213m** for the tubes used in this lab.

Using equations 4 and 5, we find a preliminary expression for the deflection, x, in terms of the magnetic field strength:

$$\frac{S^2}{2x} \approx \frac{1}{B}\sqrt{\frac{2mV_{acc}}{e}} \tag{6}$$

We cannot measure the strength of the magnetic field directly, but we can express it in terms of the current that produces it. To simplify the math, we will make another approximation. The extreme oblong rectangular geometry of the coils used to generate the magnetic field, B, means that the two "far ends" contribute relatively little. As such, the coil can be thought of as two sets of N long wires, where N is the number of turns in the coil. The magnetic field generated by a *single* long straight wire:

$$B = \frac{\mu_o I}{2\pi a} \tag{7}$$

where a is the distance from the wire to the electron beam, I is the current which is generating B, and $\mu_0 = 4\pi \times 10^{-7}$ Tesla·m/Amp.

The magnetic field produced by the current in the top wire adds to the magnetic field produced by the current in the bottom wire. The wires on the top are about the same distance from the electron beam as the wires on the bottom. Since we are treating our coils as two sets of N wires, the magnetic field is:

$$B = B_{top} + B_{bottom} = N\left(\frac{\mu_o I}{2\pi a_{top}}\right) + N\left(\frac{\mu_o I}{2\pi a_{bottom}}\right) = \frac{\mu_o N I}{\pi a} \tag{8}$$

Substituting this result into Equation 6 and simplifying, you should be able to ***SHOW***:

$$x = \frac{\mu_o N S^2 \sqrt{e/m}}{2\sqrt{2}\pi\, a}\frac{I}{\sqrt{V_{acc}}} \tag{9}$$

Procedure

The apparatus will be pre-wired, as described below. Examine the wiring, making sure that it agrees with what is described below. Once you think it is correct, call the instructor or TA over to look at it ***before*** turning any power supply on. The circuit diagram for the CRT control box (the aluminum box at your station) is seen below, in Figure 3.

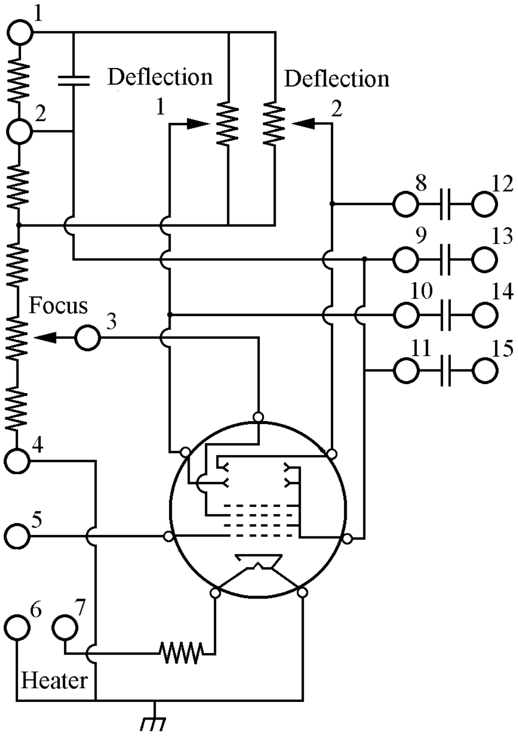

Figure 3

As before, all of the voltages needed by the oscilloscope box in this lab are produced by the HeathKit Regulated power Supply, Model IP-32. A *separate* power supply provides current to the electromagnet (Figure 4).

1. The binding post to the far left on the power supply should be connected to binding post 5 (yellow) on the aluminum box. This controls the intensity of the electron beam and can be adjusted using the knob on the left hand side of the power supply.
2. The binding post labeled common and ground on the power supply should all be connected to binding post 4 (black) on the CRT control box. Additionally, this post should be connected to the instrument ground at your station.
3. The binding post labeled 6.3 Vac on the power supply should be connected to the binding posts 6 and 7 (green) on the CRT control box.
4. The red post to the far right of the power supply should be connected to binding post 1 on the CRT control box. This controls the accelerating voltage for the electron gun. This is controlled using the knob on the right of the power supply.
5. Binding post 12 and 13 and binding post 14 and 15 on the CRT control box should be initially connected.
6. The accelerating voltage (V_{acc}) is the potential difference between binding post 2 and 5 on the CRT control box.

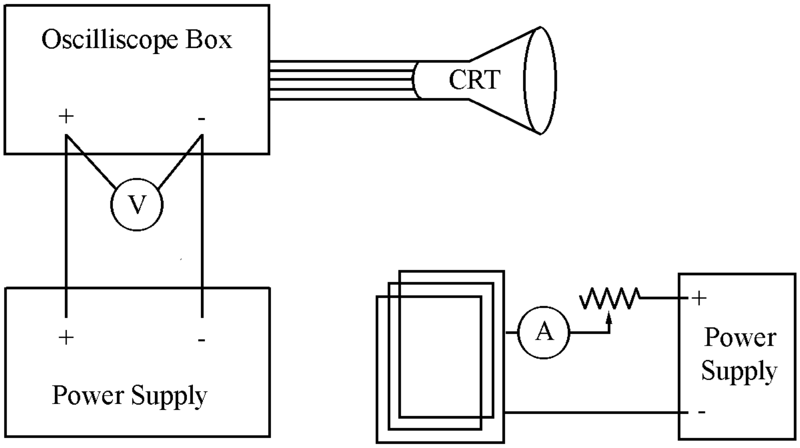

Figure 4

<u>MAKE</u> <u>SURE</u> that the ammeter is connected ***<u>in series</u>***, as shown in Figure 3, and that the CRT is *centered* horizontally in the electromagnetic coil. Once you have recorded values for *a* (remember, it's the perpendicular distance from the wires to the center of the tube), and the number of wires, *N*, have the instructor or TA make sure that everything is properly connected.

Preparing to take the data:

1. Turn the <u>400 V power supply</u> to "***standby***."
2. Turn on the <u>*coil* power supply</u>. The current knob should be set to zero initially, but both voltage knobs should be set to their largest possible value.
3. Set your meters to their appropriate settings and *then* turn them on.
4. Turn the 400 V power supply to "on" (be sure that it was on standby for 30 seconds). If you do not see a green spot on the screen, increase the accelerating voltage by turning the knob on the right of the HeathKit power supply clockwise. Once you see a green spot, adjust the focus using knob labeled ***Focus*** on the CRT control box until you observe a clear point.
5. Adjust the accelerating voltage to the point immediately before the spot vanishes. You'll be able to use accelerating voltages *between* this minimum value and 400 V.
6. Adjust the currents in the coil by adjusting the current knob on the coil power supply and watch what happens. Do not adjust the voltage knobs. When you are done, return the current in the coil to zero.

You are now ready to take the data. You will measure the deflection for at **least <u>five</u> different currents** (some positive and some negative) **and <u>three</u> accelerating voltages**. Negative currents are obtained by simply interchanging the leads to the coil. Because the brightness, focus and the location of the spot will vary with accelerating voltage, you should vary the current for fixed values for the accelerating voltage. Do try to get data with similar values, but you don't need to spend excessive time reproducing the exact current or voltage each time.

As you may have noticed, the values given the lab manual may not be identical to what your system can produce; the values listed are suggestive, not exact. As such, you may need to modify the values given to what your system can provide. With this in mind, record your data on the next page.

S = 0.213 m a =________ m N =________ e = 1.602 × 10^{-19} C

V (V)	I (A)	x_{spot} ( )	$\frac{I}{\sqrt{V_{acc}}}$
200	-3		
	-2		
	-1		
	0		
	1		
	2		
	3		
250	-3		
	-2		
	-1		
	0		
	1		
	2		
	3		
300	-3		
	-2		
	-1		
	0		
	1		
	2		
	3		
350	-3		
	-2		
	-1		
	0		
	1		
	2		
	3		

Obviously, we would like to compare our data to what the theory predicts, *i.e.* Equation 9.

$$x = \frac{\mu_o N S^2 \sqrt{e/m}}{2\sqrt{2}\pi\, a} \frac{I}{\sqrt{V_{acc}}}$$

What should you plot? What type of relationship (linear, power, exponential, *etc.*) do you expect if you plot your data this way?

Plot your data and perform the fit you proposed above. Does your data have the expected functional relationship? Discuss any unexpected results. Attach a copy of your graph to the lab manual.

There is an additional check that you can perform. In Equation 9, there is the ratio of the electron charge to the electron mass, $\sqrt{e/m}$. You should be able to extract this ratio from one of the curve fit parameters. From you fit, how can you find this ratio?

How does this result compare with the accepted value? Is this result reasonable?

Experimental Value: *e/m* =______________ C/kg

Accepted Value: *e/m* =______________ C/kg

% error =________________

Questions

1. How do your values for the magnetic field compare with the Earth's magnetic field?

2. Why does the spot vanish when the accelerating voltage is too low? Hint: Look back at the equations (5) and (9) -- this should suggest two possibilities. Discuss both possibilities.

3. Why was it important to take measurements foe the current flowing in the positive and negative direction?

4. It was assumed that the ends of the coil made negligible contributions to the magnetic field. Is this true? Justify your argument.

Initiative

Possible ideas:

1. Examine the structure of the magnetic field created by a wire, or a coil of wire. Does the magnetic field resemble what you have seen in your text and in class?

2. Carefully observe the deflections, which occur when you rotate the tube in the horizontal plane (with the coil off and the tube and its holder out of the coil frame, free to rotate on the table). Remembering that $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$, what *qualitative* statements can you make about the magnetic field in the room?

Conclusions