

Math 215 Fall 2002, Linear Algebra
Exam I

Name:

Answer all questions. 10 points per part.

1. Use the definition of linear transformation to

- (a) Show that the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with $f([x, y]) = [x^2, x + y^2]$ is not a linear transformation.
- (b) Show that the function $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with $g([x, y]) = [x - y, 3x - 3y]$ is a linear transformation.

2. Find the inverse of the matrix

$$\begin{pmatrix} 4 & 8 \\ 1 & 3 \end{pmatrix}$$

3. Using a basis:

- (a) Prove that $B = ([1, -1], [1, 2])$ is an ordered basis for \mathbb{R}^2 .
- (b) Give the B-coordinates for $[5, 2]$
- (c) Give the matrix for g with respect to this ordered basis (as in 1b, we have $g([x, y]) = [x - y, 3x - 3y]$).
- (d) Use that matrix to find the B-coordinates for $g([5, 2])$.

4. What are $\text{Ker}(g)$ and $\text{Im}(g)$?

5. What are the eigenvalues of g ?

6. What is the vector in the direction of $[3, -2]$ closest to $[1, 3]$?