Math 215, Fall 2004, Linear Algebra
Exam 1

Name:
Answer all questions.

1. Prove that Ker(L) is a subspace of the domain of L.

2. Define "the function $L : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation". Then use your definition to show that

   (a) $L([x, y]) = [-2x + 3y, -4x + 5y]$ is linear

   (b) $g([x, y]) = [2, x - 3y + 1]$ is not

3. Show that $B = ([2, 1], [1, 1])$ is an ordered basis for $\mathbb{R}^2$ and give the B-coordinates for $[-1, 2]$

4. Use $([2, 1], [1, 1])$ to get a matrix for $L([x, y]) = [-2x + 3y, -4x + 5y]$ and then use that matrix to find $L([-1, 2])$.

5. Find the eigenvalues of L.

6. What is the canonical form for the matrix for L?

7. Find the inverse for the matrix

$$\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$$

8. Prove that a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ has an inverse if an only if 0 is not an eigenvalue of T.

9. Find the vector in the direction of $[2, -3]$ closest to $[-4, 2]$.

10. Show that composition of linear transformations distributes over sum of linear transformations (that is $L \circ (S + T) = (L \circ S) + (L \circ T)$) then explain why this shows that matrix multiplication distributes over matrix sums.