Name: ______________________

Partner: ______________________

Date: ______________________

Just Torquing Around

Purpose:

In this lab, you will explore how "torque," which is the impetus for angular acceleration *differs* from "force," which is the impetus for linear acceleration. Additionally, you will explore *similarities* between the angular kinematic equations and what you have previously seen for the case of linear motion.

Background:

Consider the apparatus shown in Figure 1, showing a ***turntable*** that is initially at rest. When released, the falling mass, m_f, shown at the bottom will be acted upon by both the gravitational force (downward) and the tension in the string. A pulley of *negligible mass* is used to redirect the string in such a way that it causes a torque. At the turntable, the force due to string tension acts at some distance *from* the axis of rotation, equal to the spindle radius r_s.

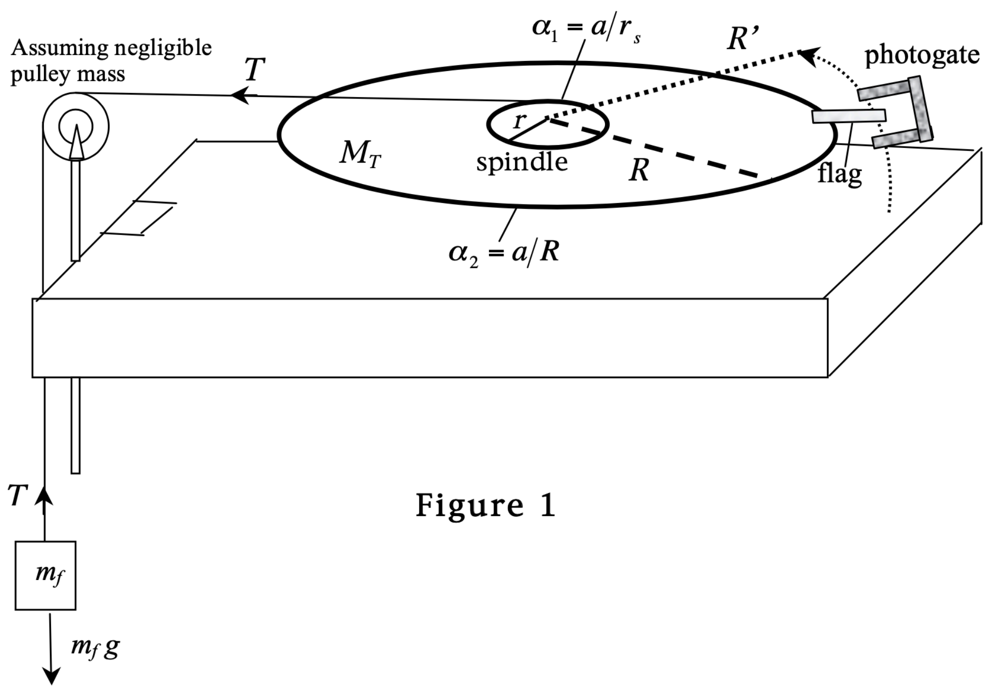


Figure 1

When examining the real apparatus in the lab, notice that the force due to string tension acts *perpendicular* to line extending from the point of contact with the spindle to the axis of rotation. Given that, the torque exerted by the force of tension is given by Equation 1.

$$\tau = I\alpha = rT \tag{1}$$

Because the mass of the pulley is much less than the mass M_T of the "turntable," it follows that the tension is approximately equal on either side of the pulley. The tension in the string can be found by applying Newton's second law to the hanging mass:

$$T - m_f g = - m_f a \tag{2}$$

Plugging this result into Equation 1 yields:

$$I\alpha = r_s \left(m_f g - m_f a \right) \tag{3}$$

The taut string ensures that everything moves rigidly *together*, meaning that the linear acceleration of the falling mass in Equation 2, a, is the *same* as the tangential acceleration of the spindle, *at the point* where the string is connected.

Separately, note that this tangential acceleration is related to the angular acceleration of the turntable by Equation 4:

$$\alpha = \frac{a}{r_s} \tag{4}$$

Plugging Equation 4 into Equation 3 yields:

$$I\alpha = r_s \cdot m_f g - r_s \cdot m_f \left(\alpha r_s \right) \tag{5}$$

From Equation 5, *you should* now solve for the angular acceleration of the disk in terms of readily measured quantities, yielding:

$$\alpha = \frac{m_f g r_s}{m_f r_s^2 + I} \tag{6}$$

If the apparatus used is such that $m_f r_s^2 \ll I$ (and we claim that *it is*), then, Equation 6 may be further simplified. In this limit, Equation 6 becomes:

$$\alpha \approx \frac{m_f g r_s}{I} \tag{7}$$

As an aside, note that Equation 7 predicts that the angular acceleration of the disk is *constant*. As a result, this experiment is the ***rotational analog*** of many constant (linear) acceleration problems that you've considered earlier in this course. You may wish to refer back to your earlier work.

We reiterate that it is the falling mass, m_f, that causes the motion, while r_s is the radius of the spindle, which the string is initially wrapped around. The factor I is referred to either as "rotational inertia" or (less transparently) as the "moment of inertia" of everything on the turntable system that spins around: in this case, that is approximately $\frac{1}{2}M_T R^2$, where R is the radius of the turntable (not the spindle).

Procedure:

In this experiment, you will calculate, from *measured* data, the angular acceleration, α, of the turntable (a *kinematic* quantity) and compare this measured value with a *prediction* calculated from Equation 7 (*i.e.*, based upon *static* quantities).

We asserted that our system undergoes a *constant* acceleration over the duration of the experiment. With this assumption, the angular acceleration is given by:

$$\alpha = \frac{\Delta\omega}{\Delta t} \tag{8}$$

Of course, there are a couple of caveats: first, our assertion that the system undergoes a *constant* acceleration will *fail* once the falling weight hits the floor, so you'll want to apply this simple model only to data taken *before* that moment. Also, our photogates aren't set up to report the *angular* velocity, though you can *calculate* the angular velocity from the reported linear velocity as long as you've **entered the correct flag width into the photogate software**, *DataStudio*, and you make use of the distance from the axis of rotation all the way out to the point where the flag intercepts the beam of infrared light utilized by the photogate. This distance is labelled as R' in Figure 1 (which you should look back at now). The linear velocities reported by *DataStudio* can be converted into angular velocities using the "**calculator function**" according to:

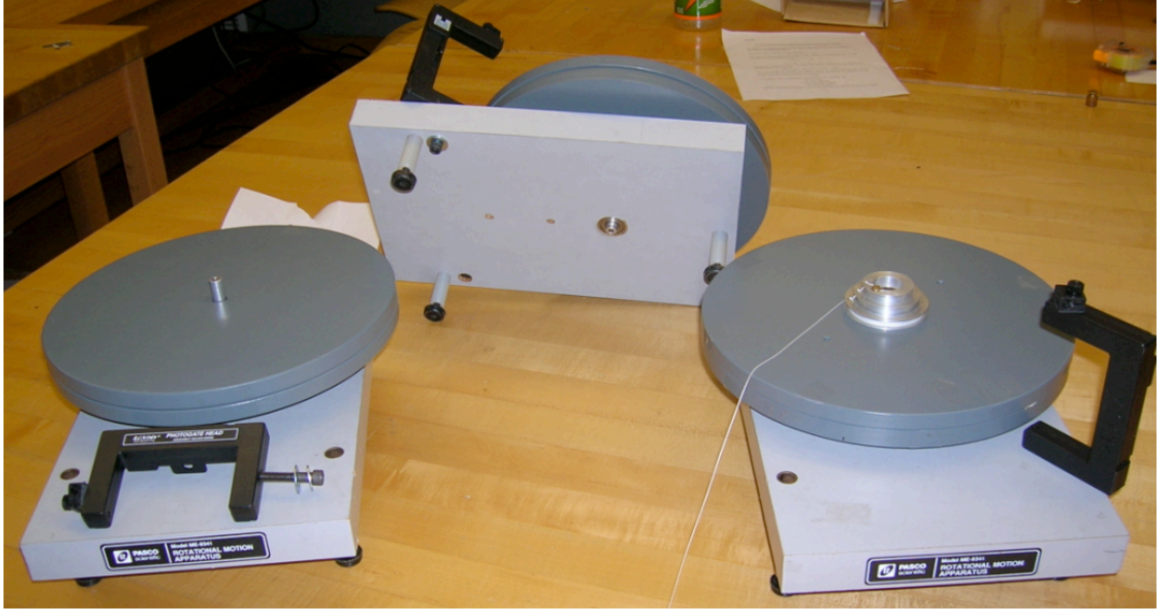
$$\omega = \frac{v}{R'} \tag{9}$$

To obtain your *experimental* value for α, you can graph the angular velocity *vs*. time and use a linear fitting algorithm, but you must thorough explain your analysis in your report, with appropriate visuals.

Quick Summary: To experimentally measure the *angular* velocity, attach a flag of width w, as seen in Figure 1. Using a PASCO photogate sensor and *DataStudio* software, you can then measure the *linear* velocity of the flag as it passes through the photo gate. — Do not forget to enter the correct flag width during the photogate configuration step. — In addition, you can use the ***DataStudio* calculator** to convert the measured *linear* velocities into their respective *angular* velocities. That leaves you in good shape for determining the *angular* acceleration, α, by fitting your data.

You should repeat the experiment for a well-chosen range of masses and then use linear fits to compare the experimental slopes you observe (for each of the masses) with the prediction given by Equation 7.

Provide an "intelligently designed" **data table** in your lab notebook (sloppiness and lack of clarity and/or organization will get you penalized). You might wish to consider your first attempts to be *rough drafts*.



Picture 1: Attaching the photogate

Because it is imperative that the photogate position remains fixed with respect to the axis of rotation, it should be attached to the apparatus as shown above, with the aid of a $1/4 - 20$ bolt. At the left above, you can see that such a bolt can be directly threaded into the bottom of a photogate. In the example shown in the middle of the picture above, you are provided with a bottom view of the attachment. At right above, you see the photogate in an orientation appropriate to use in the experiment. — A fully assembled apparatus, with the flag attachment, is also shown below, in Picture 2 (the bottom photograph):

Picture 2: Assembled system (including a useful set tools for this activity)

Act II: Conservation of Energy (you already have the data!)

Again, consider your experimental arrangement. We can easily apply the principal of conservation of energy, as the falling mass, m_f, descends a distance Δh:

$$\frac{1}{2}\left(I + m_s r_s^2\right)\omega^2 + \frac{1}{2} m_f v^2 = m_f g \Delta h$$

Discuss how the equation above *makes sense* and, from it, show that the equation below must *follow*. [*Hint:* how are v and ω related?]

$$\omega^2 = \frac{2 m_f g}{I + \left(m_s + m_f\right) r_s^2} \cdot \Delta h$$

That is,

$$\omega^2 \approx \left[\frac{2 m_f g}{I}\right] \cdot \Delta h$$

Why would we want to express the relationship in this fashion?
What falling masses can be used in order for the preceding approximation to be valid?
[You do NOT have to use the approximation!]

Discuss the *linearization* method as it pertains to this analysis. That is, what's the advantage of using a plot of ω^2 vs. h (**which you should now create!!**)?

Carefully describe how you can relate the circumference C of your spindle to successive changes in heights, each time the turntable makes another revolution. Then, connect these height changes to their respective angular velocities (already obtained prior to Act II).

Does the slope of your plot for Act II agree with what you would predict, based upon values obtained from static measurements for $2 m_f g / I$? — If there is reasonable agreement, what does this prove?

Questions:

1. Are your experimental results consistent with your expectations? Discuss.

2. The mass of the three-tier spindle is approximately 100 grams, however we neglected its moment of inertia in the calculations. Show using calculations, that this approximation is justified.

3. In going from Equation 6 to 7, we claimed that the approximation $mr^2 << I$ is valid. Is this approximation justified for the apparatus that ***you*** used (*i.e.*, calculate the quantities mr^2 and I, and compare)?

Initiative:

Possible ideas:

1. List and describe, in detail, all of the torques acting on the disk. Did we neglect any that should have been considered? Can you explore these?

2. Determine if we are justified in using *average* angular velocities, when - strictly speaking - our calculation of a calls for *instantaneous* values. That is, is there anything that could be done to improve the values of ω and ω_0? (Hint: see the "initiative page" from the lab on the Work-Energy Theorem).

3. Discuss whether mechanical energy is conserved in this experiment. If not, what fraction is lost? Where is the energy lost?

4. Try activities suggested in this video: iOLab Activity on Angular Acceleration Moment of Inertia

5. Try activities on Pivot Interactives, regarding a Disk Accelerated by Hanging Weight.

Conclusions: