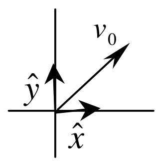
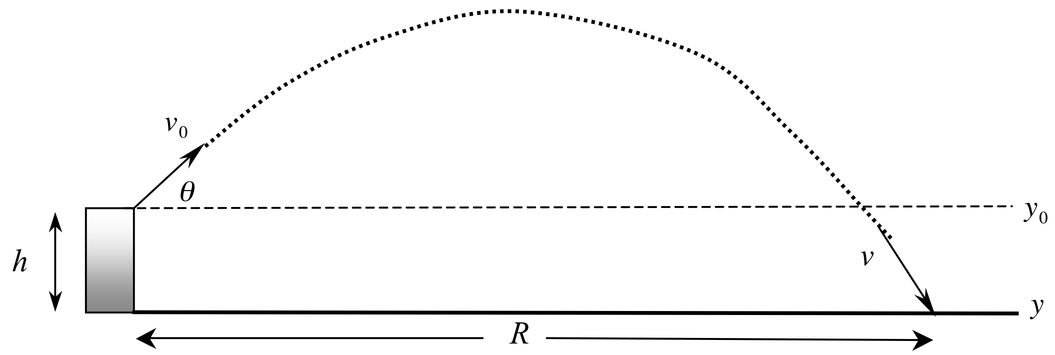
ACTIVITY II

Projectile Fired from an Elevated Vertical Position

Consider a simple projectile (no friction) launched from an elevated height as shown in the figure below.

coordinates





<u>Solution:</u>

The final y-velocity: Solving for the y-component of the final velocity we can use the kinematic relation

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y\left(y - y_0\right),$$

where $a_y = -g$, $v_{0y} = v_0 \sin\theta$ and $y - y_0 = -h$. We should take the "-" root (Why?)

<u>Show that the final y-component of velocity is (and explain the negative sign)</u>

$$\boxed{v_y = -\sqrt{v_0^{\,2}\sin^2\theta + 2gh}} \longrightarrow \text{SHOW!}$$

The time of flight: The time the projectile is in the air can be obtained from the kinematic relation

$$v_y = v_{0y} + a_y t.$$

<u>Show that the total time of flight t_T is a function of launching angle θ and firing position h an may be expressed as</u>

$$\boxed{t_T = \frac{v_0 \sin\theta + \sqrt{v_0^{\,2}\sin^2\theta + 2gh}}{g}} \longrightarrow \text{SHOW!}$$

The range: We may compute the range from the kinematic relation

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2.$$

<u>Show that the total range R_T is also a function of launching angle θ and firing position h and can be computed from</u>

$$\boxed{R_T = v_0 \cos\theta\left[\frac{v_0 \sin\theta + \sqrt{v_0^{\,2}\sin^2\theta + 2gh}}{g}\right]} \longrightarrow \text{SHOW!}$$

The analysis, if properly presented, will suffice as a good theory section in your report.

Procedure

Please examine the construction (carried out by two IWU profs) of the "carriages" on which the spring guns are mounted. Why is it important that the pivot axis be located through the center of the steel ball when the piston is extended to its firing position? You should also discuss the compass and/or torpedo level arrangement. Generally, it is a good idea for physicists and engineers to examine how an apparatus functions and also to have some idea of its intrinsic accuracy.

I) First, you are to *calibrate* the spring gun by determining the "muzzle velocity", $v_0 \equiv v_m$. This can be achieved by firing the gun vertically and measuring the maximum height obtained by the projectile. Repeat the process several times in order to obtain a good average value for H_{max}. You should carefully align and ensure that the results are reproducible. Using the kinematic relation

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y(y - y_0)$$

show that

$$\boxed{v_{0y} \equiv v_m = \sqrt{2gH_{max}}}$$

You must also include a well-drawn figure with the appropriate initial conditions indicated.

II) Choose an appropriate range of angles and then acquire data for the distance the ball travels as a function of launching angle. Explain how you are measuring all the relevant heights, distances, and other parameters. After recording your data in a **well-constructed table** graphically analyze your data. Should we even point out that some discussion is also needed?
The "formula entry" must simulate the theoretical expression you have already derived, *i.e.*

$$R_T = v_m \cos\theta \left[\frac{v_m \sin\theta + \sqrt{v_m^{\,2} \sin^2\theta + 2gh}}{g} \right],$$

Some brief comments on how to perform a fit using this formula in Igor Pro: Under Analysis choose "Curve Fitting …" and go to the "Function and Data" tab. After setting the correct X/Y data columns (called "waves" in Igor), choose to define a new function and give it a name. Enter some strings such as "Vm" and "h" (without quotes) as the fit coefficients and enter "x" as the independent variable. Then type in the fit expression using "^2" for squaring and "sqrt()" for taking the square root. Your fit expression may look like

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f(x) = (Vm*cos(x*pi/180)/9.8)*(Vm*sin(x*pi/180)+sqrt((Vm*sin(x*pi/180))^2 + 2*9.8*h))
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Notice that pi/180 is needed to convert degrees to radians, which is what Igor expects.

You should try two types of fit to the data:

1. Where v_m is the measured muzzle velocity and and h is the measured launching height. In this case there will be no fit parameters. You can do this in Igor Pro by going to the "coefficients" tab in the Curve Fitting menu. As initial guesses enter the measured values of v_m and h and select the "Hold" option on both so that they will not be altered.
2. Where h is the measured launching height but you are allowing v_m to be a fit parameter. To do this just uncheck the "Hold" option for v_m.
3. Optional – Initiative: leave both h and v_m as fit parameters and explain the results

You should at least compare the measured value of v_m and the one extracted from the fit and explain any difference between the two. And you can do more to show your initiative. For example, what firing angle gives the longest range, what are the "other" sources of error (i.e. excluding measurement error).

SIMULATION SHOWN FOR YOUR BENEFIT

Here is a table with calculations carried out in 15-degree increments:

Launch angle (degrees)	Range (w/ h=1m)
0.0000	2.8000
15.000	3.8583
30.000	4.6596
45.000	4.7485
60.000	3.8998
75.000	2.2001
90.0 0	0

The value of *h* was set to 1 m here and v_m was extracted from the fit. Notice the properly labeled axes and the error bars on the plot. Ask instructor/TA for help if you need help with these.