Name: ______________________

Partner(s): ______________________

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Date: ______________________

Work-Energy Theorem

Purpose:

Everyone says, "Energy is conserved." We decided early on that our decisions about the validity of every hypothesis must, ultimately, appeal to experiment. Here, then, is your chance to take on "the big one": to investigate the validity of the work-energy theorem.

Theory:

If an amount of work, *W*, is done on a system, then the kinetic energy, *K*, of the system changes. The *change* in the kinetic energy is equal to the amount of work done. Stated quantitatively:

$$W = \Delta K = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} \qquad (1)$$

Equation 1 is a statement of the work-energy theorem and is the foundation of much of our science. As such, questions of its validity are certainly worth your consideration.

Procedure:

The experimental setup is depicted in Figure 1. The inclined plane is an air track and the mass M is meant to represent the levitated cart. The mass m is a hanging mass, which will provide the force that does work on the levitate cart. As before, ***don't drop the cart*** (and if it is accidentally dropped, ask your instructor to inspect it carefully). Also, **adjust the pulley so that the string passing over the air track is parallel to its surface**.

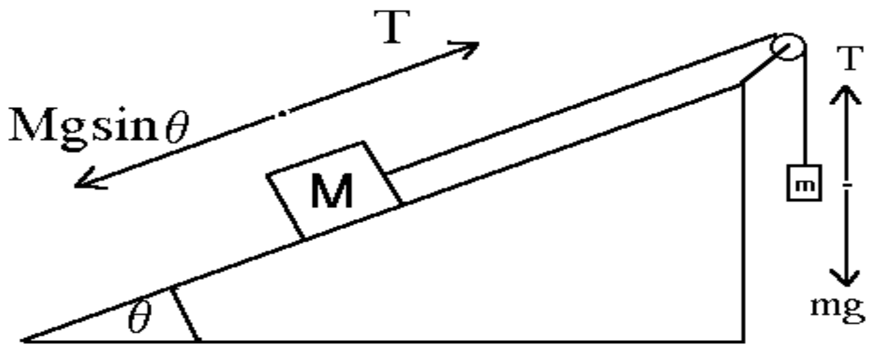

Figure 1: Experimental Setup with force-body diagram

As a preliminary step, we need to calculate the work done on M. The calculation is simplest if we ensure that the net force acting on the cart is parallel to the displacement (that's why we just asked you to adjust the pulley).

Applying Newton's second law to each of the two masses and defining the motion to the right of the figure to be *positive* yields the following equations:

for M: $$T - Mg\sin\theta = Ma \tag{2}$$

for m: $$mg - T = ma \tag{3}$$

From this, we find (and hopefully you do, too) the acceleration of the levitated cart to be:

$$a = \frac{(m - M\sin\theta)}{M + m}\,g \tag{4}$$

Since the net force acting on the cart's mass is $F = Ma$, the work this force does in displacing the mass a distance s is $W = Mas$. Using the expression we found above for the acceleration, we find the work done on the levitated cart.

$$W = \frac{M(m - M\sin\theta)}{M + m}\,gs \tag{5}$$

This is the work that the net, unbalanced force will perform, if the unbalanced force causes a displacement s of the cart up the inclined air track. Note that the work, W, can be calculated from easily measured quantities.

If we can now determine a procedure for calculating the change in the kinetic energy of the cart, we can determine if the work-energy theorem is valid. From previous labs, we know that there are a number of different methods to measure the velocity of an object. One such method involves using a photogate. The cart's velocity is measured by having the photogate measure note the length of time, Δt, that the cart blocks the beam. Having measured this time, Δt, and the length of the (flag) part which passes through the photogate, Δx, the software can tabulate the average velocity ($\Delta x/\Delta t$) which the cart had while passing through the photogate.

As always, if it is possible, we like to simplify things in physics. One way to simplify the determination of ΔK is to make the initial kinetic energy zero; the change in the kinetic energy of the cart will then be simply the kinetic energy of the cart after it has been displaced a certain distance s. If the cart starts at rest, the initial velocity is zero. By placing the photogate a distance s from the cart's initial position, we will measure the velocity of the cart after it has traveled a distance s. Knowing the mass of the cart, we can calculate the change in the kinetic energy. The all-important comparison between W and ΔK is all that remains.

You might note that we are measuring the *average* velocity of an accelerating object, when we really want to know the instantaneous velocity after the cart has traveled *exactly* a distance of s. Is this approximation valid? Discuss any limitations and how you will proceed to minimize these limitations.

Data:

m = ____________________ kg

M = ____________________ kg

$\sin(\theta)$ = ____________________

A sketch might clarify your measurement of $\sin(\theta)$:

As always, you will want to take multiple measurements to increase the validity of your conclusions. As such, you will want to measure the change in kinetic energy for a number of distances, *s*.

What factors influenced your choices of *s*?

Your data can be recorded below.

Trial	s ( )	Δx ( )	Δt ( )	v ( )	v^2
1					
2					
3					
4					
5					
6					

In the table below, calculate the work done on the cart and the change in the cart's kinetic energy.

Trial	W ( )	ΔK ( )	% Difference
1			
2			
3			
4			
5			
6			

$$W = \frac{M(m - M\sin\theta)}{M + m} gs \qquad \Delta K = \frac{1}{2}Mv^2$$

Is the work-energy theorem valid?

Over the last few labs, we have made a point of emphasizing how it is important to perform multiple independent experiments when testing a "new" idea. In this case, you can use the kinematic equations as a sort of "check."

How is velocity related to the displacement?

Plot your velocity and displacement on the appropriate set of axes. An appropriate fit of this data should allow you to extract a value for the acceleration of the cart. Describe what you plotted and why. Attach a copy of your plot and fit.

From your fit, what is the acceleration of the cart? How does this value compare with what is predicted in Equation 4?

Questions:

1. In order for the work-energy theorem to have meaning, work ($\boldsymbol{F} \cdot \boldsymbol{d}$) and kinetic energy ($mv^2/2$) should have the same units. Show that they do.

2. Does work cease to be done on the cart when the small mass comes to rest on the ground? Explain.

3. Are you justified in treating the inclined air track as a frictionless inclined plane?

Initiative:

Conclusions: