Name: ____________________

Partner(s): ____________________

____

Date: ____________________

Ballistic Pendulum

1. *Purpose:*

Primarily, this lab is included just because we like the word "ballistic." A secondary purpose of this experiment is to examine the principle of ***conservation of momentum***, using something called a "ballistic pendulum."

2. *Background:*

We used to teach this lab with live ammunition. Students would fire their guns at a wooden block suspended by a rope (*i.e.*, a simple pendulum). By measuring how high the pendulum would swing, one could determine the initial velocity of the bullet. It's unclear why we stopped using real rifles. Today, the basic idea remains the same: a projectile is fired into a (hollow) pendulum bob and remains embedded in it, as seen in Figure 1.

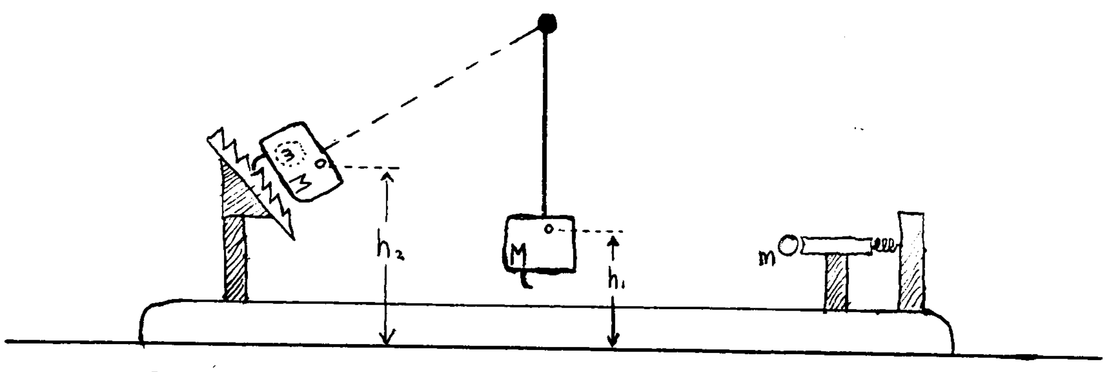

Figure 1: The ballistic pendulum

We have seen, in previous labs, that it is generally a good idea to perform multiple independent experiment. This is especially true when you are looking at a new idea, when you can use what you have already found to be valid as a sort of check. As such, you will perform two experiments and compare the results.

2.1 *Experiment 1: Conservation of Momentum*

When the projectile collides with the pendulum bob, the projectile remains embedded in the pendulum bob - a completely inelastic collision. Using the variables defined in Figure 1, applying conservation of momentum to the collision yields:

$$mv = (M + m)V \tag{1}$$

After the collision, the pendulum bob will swing upward until all of its kinetic energy is converted into gravitational potential energy. If the vertical distance traveled by the pendulum bob is h, conservation of energy tells us that:

$$\frac{1}{2}(M + m)V^2 = (M + m)gh \tag{2}$$

As we discussed earlier in this lab, you will be comparing the initial velocity of the bob, as determined using two different methods. In the space below, use the above results to find an expression for the velocity of the projectile immediately before the collision in terms of easily measurable quantities

You should find Equation (3).

$$v = \left(\frac{M + m}{m}\right)\sqrt{2gh} \tag{3}$$

where v is the velocity of the projectile before the collision, g is the acceleration due to gravity, h is the vertical displacement, M is the mass of the pendulum bob and m is the mass of the projectile.

2.2 *Experiment 2: Kinematics*

Of course, for the projectiles fired from the spring-loaded guns we use these days, you never really needed to use the pendulum to find the initial velocity, right? If you fire the gun and measure how far the projectile travels, you can use the kinematic equations to determine the initial velocity of the projectile. In a previous lab, we determined that these equations are valid. This means that we can test the validity of conservation of momentum by comparing the initial velocities found using the kinematic equations and conservation of momentum.

Suppose that the projectile is fired *horizontally* a distance, y, above the floor with an initial velocity, v_o. According to the kinematic equations, what are the horizontal and vertical positions of the projectile when it hits the ground? Be sure to clearly identify any variables that you use. Sketches are always useful.

Use these results to find the initial velocity of the projectile in terms of easily measured quantities.

You should arrive at Equation (4).

$$v = \frac{x}{\sqrt{2y/g}} \tag{4}$$

where v is the initial velocity of the projectile, g is the acceleration due to gravity, x and y are the horizontal and vertical displacement, respectively.

3 Procedure:

You will determine the initial velocity of a projectile, steel ball shot with a spring loaded gun, using each of the methods outlined above.

3.1 Part 1: Momentum and Kinetic Energy after the Collision

- Before using the ballistic pendulum, as described in Method 1, make sure that the pendulum bob is screwed snugly into its axle (there is a knurled knob on the underside of the top support). Then, simply fire the steel ball into the hollow pendulum bob.
- After the collision the pendulum will be brought to rest at its highest point by a set of ratchet teeth mounted on the base of the instrument. This is a *convenience*, which allows you to take your time when measuring the distance, h, that the pendulum was raised. Several values of h should be averaged. Note that h is simply the difference in the measured values of h_2 and h_1 in Figure 1. These heights should be measured from the tabletop up to the center of mass of the system, which is indicated by a *red dot* on the pendulum bob.
- Measuring the masses of the two colliding bodies will then supply all of the information needed to calculate the initial velocity of the projectile using Equation (3).
- Record any relevant data below.

M = ________________ m = ________________

Trial	1	2	3	4	5
Height, h_2 ( )					

Average h_2: $\bar{h}_2$ = ________________

h_1 = ________________

$h = \bar{h}_2 - h_1$ = ________________

- Calculate the velocity of the projectile after the collision:

v_f = ________________

- Calculate the kinetic energy and momentum of the projectile after the collision:

$K_f = -\Delta U$ = ________________

p_f = ________________

3.2 *Method 2: Momentum and Kinetic Energy before the Collision*

- Before using the ballistic pendulum, as described in Method 2, make sure that the pendulum bob is *out of the way* so that you can fire the projectile across the room. There is no need to unscrew the pendulum from its support, simply raise it until the ratchet teeth engage the bob.

- **The first time you test the range, please make sure you aren't likely to hit anything other than the floor**. It may take a shot or two until you have a feel for where the ball is likely to land. Once you have a pretty good idea of where the ball will land, place a piece of carbon paper (carbon side *up*) on the landing spot and tape a white sheet of paper on top of the carbon.

- When the ball lands, it will leave a mark. By measuring the location of the marks, you can determine the range of the projectile.

- As always, several measurements should be made. Be sure that you use the same spring compression as you did in the previous part of this lab.

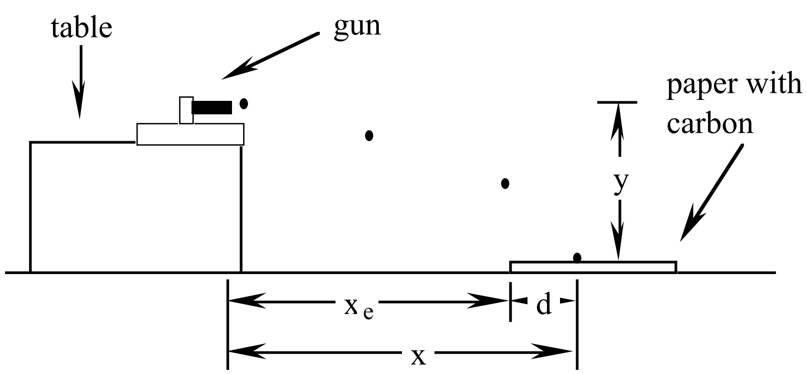

Figure 2

As is indicated in Fig. 2, the actual **range,** x, is composed of two pieces: first there is x_e, the horizontal distance from the launch point to the edge of the paper and second, d, from the edge of the paper to the center of the shot distribution. Thus, $x = x_e + d$.

Feel free to ask for any equipment that you need to make these measurements.

x_e = ________________ y = ________________

Trial	1	2	3	4	5
d ( )					

$\bar{d}$ = ________________

$x = x_e + \bar{d}$ = ________________

- Calculate the velocity of the projectile before the collision:

v_o = ________________

- Calculate the kinetic energy and momentum of the projectile before the collision:

K_o = ________________

p_o = ________________

4 *Results:*

v_o	= ________________	v_f	= ________________
p_o	= ________________	p_f	= ________________
K_o	= ________________	K_f	= ________________

- How do the initial and final velocities compare? What is the percent difference?

- Is momentum conserved? Why or why not?

- Is kinetic energy conserved? Does this result make sense? (Hint: If kinetic energy was not conserved, where is the missing energy? If energy was lost, where did it go? If it was gained, where did it come from?)

5. *Questions:*

i. Discuss how the two conservation laws of mechanics (*i.e.*, conservation of momentum, and conservation of energy) were used in the ballistic pendulum method.

ii. Compute the fractional loss of energy involved in the inelastic collision: $(\% \ loss \ = \ \dfrac{K(final) \ - \ K(initial)}{K(initial)} \ x \ 100)$. Does this result make sense?

iii. If the ball had bounced off the pendulum with a smaller fractional loss of energy, would the pendulum have risen to a higher height, or not quite as high? By what factor?

iv. Calculate, theoretically, the ratio of the kinetic energy of the system before the collision to the kinetic energy of the system after the collision, *i.e.* $K_{final}/K_{initial}$. Eliminate all other variables from your answer except for m and M.

6. Initiative:

<u>Possible ideas:</u>

- ❖ Investigate whether or not the range measurements exhibit a normal (or Gaussian) probability distribution. Give a graph with mean and standard deviation. (Refer back to the lab on "Statistical Thinking")

- ❖ Which of the two methods for determining the momentum of the ball do you think is the more accurate and why? (Consider *systematic* errors)

- ❖ Determine the moment of inertia and then the radius of gyration of the pendulum and bullet combination. (See your text for discussion of a "physical pendulum", as opposed to the "simple pendulum" treated earlier)

- ❖ Estimate the range of a 22-caliber bullet. (Explain your work).

7. Conclusions: