Name:
Partner(s): $\qquad$

Date:

## One Dimensional Motion

## 1. Purpose:

This lab has a dual purpose. We want you to become familiar with the operation of a linear air track and recognize this as another instance of the cultural preference that physicists have towards simplicity and minimalism. You will then use it to investigate the motion of a body subjected to a constant force and discover, experimentally, for this situation, the functional relationship between distance, velocity, and time.

## 2. Background:

> Required for this lab:
a) Familiarize yourself with the basics of Igor Pro graphing software
b) A good understanding of the lab reading material provided here.
c) Science Workshop ${ }^{\mathrm{TM}}$ interface box (must be turned on)
d) DataStudio ${ }^{\text {TM }}$ software program (started after the Science Workshop ${ }^{\text {TM }}$ box)
"Kinematics" is the study of motion, as opposed to "dynamics", where one attempts to understand the "cause" of motion. In this experiment you will analyze one-dimensional motion and determine the equation of motion, expressed in the form:

$$
\begin{equation*}
x_{1}=f(t) \tag{1}
\end{equation*}
$$

where $\boldsymbol{x}_{\boldsymbol{t}}$ is the location of the object at time $\boldsymbol{t}$.
To study one-dimensional motion, you will use a cart and an air track. An air track is a nearly frictionless device, which can be used to study motion under ideal conditions. Carefully inspect all parts of the air track and note their function. It is a delicate instrument and should be treated as such. In particular, don't drop the carts! If a cart is dropped, contact your instructor and ask them to inspect it carefully. With the blower on, inspect all the air holes to ensure that they are open.

In practice, we can never (or can we?) achieve a perfectly frictionless environment; even a levitated cart set in motion will eventually come to rest. Thus, you should investigate how good an approximation it is to consider the air track frictionless.

The simplest example of motion is an object moving at constant velocity, $v_{o}$, in a straight line. In this case, the change in an object's position after a period of time, $\Delta t$, is trivial:

$$
\begin{equation*}
\Delta x=v_{0} \Delta t \tag{2}
\end{equation*}
$$

When the object is accelerating (i.e. the air track is inclined, or there is substantial friction, or there is a big draft of air, etc.) this relationship will be more complex.

The next-simplest case is if the acceleration $\boldsymbol{a}$ is uniform (i.e. does not vary over the time of the experiment); then the change in an objects position is:

$$
\begin{equation*}
\Delta x=v_{0} \Delta t+\frac{1}{2} a(\Delta t)^{2} \tag{3}
\end{equation*}
$$

where $v_{o}$ is the initial velocity of the object at the beginning of the interval $\Delta \boldsymbol{t}$ and $\boldsymbol{a}$ is the uniform acceleration that the object is subject to. Because the acceleration is constant, the velocity, $v_{f}$, of the object after time, $\Delta t$, satisfies:

$$
\begin{gather*}
v_{f}=v_{0}+a \Delta t  \tag{4}\\
v_{f}^{2}=v_{0}^{2}+2 a \Delta x \tag{5}
\end{gather*}
$$

You can derive Equation (5) by squaring Equation (4) and substituting in Equation (2).

If the acceleration, $\boldsymbol{a}$, is itself changing with time, the equations of motion are more complex. For now, you will study the motion of the cart down the inclined air-track for which equations (3) and (5) will probably provide a reasonably good description.

## 3. Procedure:

The setup is depicted in Figure 1. Data will be taken using two photogates, shown in the figure as $\mathrm{G}_{0}$ and $\mathrm{G}_{1}$. The air track will be elevated at one end, and you must determine the resultant angle of inclination.


Figure 1

In the first part of this lab, you will measure the duration of time, $\boldsymbol{t}$, that it takes for the air cart to travel a distance $\Delta \boldsymbol{x}=\boldsymbol{x}(\boldsymbol{t})-\boldsymbol{x}_{\boldsymbol{o}}$ on the air track. The distance $\Delta \boldsymbol{x}$ is the distance that the cart travels between the two photogates. You will repeat this six times for six different distances, $\Delta \boldsymbol{x}$.

If the velocity at the initial photogate, $v_{o}$, and the acceleration remain the same during each run, then you should have the form of Equation (3). The acceleration will be constant if the angle of inclination, $\theta$, remains constant and any frictional forces do not depend on the location or speed of the cart. Additionally, if you start the cart from rest at the same point on the air track and the position of the first photogate remains fixed during each measurement, the velocity at the initial photogate will remain constant. Different distances $\Delta x$ can then be obtained by varying (only) the location of the second photogate.

Before you can make your first computerized measurement, ensure that your photogates are connected to a Science Workshop ${ }^{\text {TM }}$ interface box, and that the box is turned on. After that, you can initialize the DataStudio ${ }^{\mathrm{TM}}$ control software and select "Create Experiment." This brings up a graphical representation of the interface box, showing its various frontpanel connectors.
$>$ Click on the picture of the connectors your photogates are connected to, and use the resulting dialog box to let the control software know that that you do indeed have photogates connected. Do this for each of the two inputs that you are using.
$>$ Now that you have configured the inputs, select any measurements you would like the control software to retain.
$>$ Click on the "third tab" generated and enter the width of the paper "flag" attached to your cart that will actually trigger the photogates.
$>$ You also need to enter the separation between photogates. (You will want to update this entry whenever you change the set-up.)

By inspection, you might expect that the acceleration, $\boldsymbol{a}$, depends upon the angle of inclination, $\theta$, of the air track.
$>$ In the space below and on the next page, draw a force-body diagram of the cart and apply Newton's $2^{\text {nd }}$ Law to find the acceleration of the cart in the direction of motion.

You should find that the acceleration is given by Equation (6).

$$
\begin{equation*}
a=g \sin \theta \tag{6}
\end{equation*}
$$

$>$ Keeping $\theta$ fixed is easy: place a block of fixed height $\boldsymbol{h}$ under one end of the airtrack. (Some air-tracks only have bumpers at one end; make sure that the lower end of your air track does have one!) The first photogate must be kept stationary, perhaps $15-20 \mathrm{~cm}$ from the high end of the track.
$>$ You should measure seven "reasonably spaced" data points covering the entire length of the air track.
$>$ For each distance $\Delta \boldsymbol{x}$ in Table 1, find the time interval $\Delta \boldsymbol{t}$ by averaging over six trials.

## 4. Data:

## Determine the angle of inclination:

$$
\begin{gathered}
\boldsymbol{m}_{\text {cart }}= \\
\boldsymbol{h}= \\
\boldsymbol{L}= \\
\theta=\tan ^{-1}(h / L)= \\
\hline
\end{gathered}
$$

> Consult the drawing on the previous page to make sure that you measure the proper $h$ and $L$. The most common error made is an incorrect determination of the angle of inclination!

## Measuring the Time of Travel for Various Distances:

Be sure you are recording, here, the time between Gates $1 \& 2$, and not some other time!!

| Trial | $x_{0}$ (Initial Gate) | $x_{\mathrm{f}}$ (Final Gate) | $\Delta t$ (Time) | $\Delta x$ (Displacement) |
| :--- | :--- | :--- | :--- | :--- |
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## 5. Analysis:

## Part 1:

$>$ Enter your data, $\Delta \boldsymbol{x}$ and $\Delta \boldsymbol{t}$, into your graphing program, IGOR Pro. From the previous discussion, we know that the relationship should be modeled by Equation (3), but graphing programs tend to supply more generic looking fitting functions, so we often have to re-phrase the equations associated with our physical models so as to match up with the syntax of the graphing software: here, we expect that your plot of $\Delta x$ and $t$ should appear to resemble a polynomial of second order, as seen in Equation (7).

$$
\begin{equation*}
y(x)=a_{0}+a_{1} x+a_{2} x^{2} \tag{7}
\end{equation*}
$$

Here, $y$ and $x$ refer to the $y$-axis and the $x$-axis of a graph, not physical quantities, and $a_{0}, a_{1}, a_{2}$ represent the adjustable coefficients of the polynomial. They are adjusted by your curve-fitting software in order to make the fitting curve best approximate your data. It is then up to you to decide whether the resulting polynomial is a physically reasonable (or "meaningful") fit to your experimental data:
> Perform the fit and record the "best-fit values" output by your graphing software for the polynomial coefficients:

> Attach a copy of your graphed data, including the fitted line.

Again, once your graphing software "spits out" a fit to your data, it is then up to you to decide whether the resulting polynomial is physically reasonable (or "meaningful"): comparison of equations (3) and (7) tell us that, according to our physical model, we expect $a_{0}$ to be $0, a_{1}$ would correspond to $v_{o}$ and $a_{2}$ would be $g \sin \theta / 2$. You have already computed a theoretical value for $\boldsymbol{a}$ using Equation (6). You should also compute a theoretical value of $v_{o}$, by using Equation (3). With these in hand, complete the table below:

|  | Experimental | Theoretical | \% Error |
| :---: | :---: | :---: | :---: |
| acceleration, $a(\quad)$ |  |  |  |
| initial velocity, $v_{o}(\quad)$ |  |  |  |

* Does your fitted value of $a_{o}$ agree with what you expect? Comment.
* Does your fitted value of $a_{1}$ agree with what you expect? Comment.
* Does your fitted value of $a_{2}$ agree with what you expect? Comment.


## Part II:

We can also study kinematics by measuring the initial velocity, $v_{o}$, and final velocity, $v_{f}$. When you measured the time that the cart spent between the photogates, the software also measured the time required for the cart to pass through each photogate, $\Delta t_{0}$ and $\Delta t_{\mathrm{f}}$. If the length, $\Delta L$, of the flag that passed through the photogate is sufficiently small, the quantities $\Delta L / \Delta t_{0}$ and $\Delta L / \Delta t_{\mathrm{f}}$ provide good approximation to the instantaneous velocity of the cart as it passes through each photogate.

The choice of the length, $\Delta L$, is a compromise between two conflicting needs. If $\Delta L$ is too small, then the transit time is too small to measure precisely and the percentage error in determining $v(t)$ becomes unacceptably large. On the other hand, if $\Delta L$ is too large, the measured speed is not a good approximation to the instantaneous speed. Be careful in determining your value of $\Delta L$.

Explain your method in the space below.

If you were careful to include the necessary measurements when taking your data earlier, you do not need to make any additional measurements now. (Aside: reading the lab write-up ahead of time does save you time in lab!) The table below will guide you through what is needed to calculate the acceleration using Equation 4.

$$
\Delta L \quad=
$$

| $\Delta x$ <br> (Displacement) | $\Delta t_{0}$ <br> (Time, <br> Gate 1) | $\Delta t_{\mathrm{f}}$ <br> (Time, <br> Gate 2) | $v_{0}$ <br> (Velocity, <br> Gate 1) | $v_{\mathrm{f}}$ <br> (Velocity, <br> Gate 2) | $a_{\text {cal }}$ <br> (Accel) |
| :--- | :---: | :---: | :---: | :---: | :---: |
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$$
a_{\text {average }}=
$$

## 6. Questions:

i. What is the magnitude of the force exerted on the air cart by the air from one of the holes in the air track (i.e., the force lifting the cart up)?
ii. Is the assumption that the air track is frictionless valid? Support your answer using the data you took in this lab.

## 7. Initiative:

Possible ideas:
Design an experiment to determine the extent of frictional effects on the air track. Do this experiment and present your data and analysis.

## 8. Conclusions:

