Name: ____________________

Partner(s): ____________________

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Date: ____________________

Measurement and Random Error

1. Purpose:

In this lab, you explore the nature of random error. You should also develop an intuitive feel for a few relevant statistical terms, which are useful in describing measured values.

2. Introduction:

The scientific validity of any hypothesis must, ultimately, appeal to experiment. The implicit goal of our experiments, then, is to extract truths about the physical world. However, the understanding of "truths" according to a 16th century physicist is different from those of a late-20th century physicist, which may in turn differ from those of a 21st century physicist. Our framework for the understanding of nature evolves over time. This lab introduces a way to deal with gauging the precision, but not necessarily accuracy, of a result, *i.e.*, *random* errors but not necessarily *systematic* errors.

This lab introduces you to the elements of recognizing, displaying, and quantifying random variations of different kinds. Such variation is omnipresent. Rigid determinism is rare in science, as well as in other human enterprises. If you repeat the same measurement over and over, it is quite likely that your result will not *exactly* reproduce itself. You might try to eliminate the effect of random variations between trials by taking your final result to be the average over many trials - but can you quickly estimate whether you have taken *enough* data to average out such effects? Can you estimate by how much your final result might shift around if you continued to take more data? This lab is intended to give you some notion of inference, the drawing of conclusions from uncertain data.

Due to unnoticed variation in a sample of measurements, repeated measurements of a variable $\boldsymbol{x}$ do not give the same answer. Unlike popular terminology, for a scientist to say that $\boldsymbol{x}$ is a randomly fluctuating variable does not imply that $\boldsymbol{x}$ takes on values "completely arbitrarily" and that "nothing" can be said about the outcome of attempted measurements of $\boldsymbol{x}$.

The most frequent methods of representing, quantifying and communicating variations of a single random variable are given below:

a. ***literal***: making tables (complete, but tiring to read)

b. ***graphical***: histograms (highly visual)
c. ***mathematical***: computing the mean, standard deviation, and other "moments" of the distribution - simple numbers which convey the information contained in the histogram succinctly.

You will use all three methods in this lab.

Frequently, we will use two parameters to characterize a distribution: the mean and the standard deviation. If you make *n* measurements of some quantity and we denote these measurements by x_i, where i= 1,2,...*n*, then the mean and standard deviation of your data are defined by Equations (1) and (2), respectively.

$$\bar{x} = \frac{\sum x_i}{n} \tag{1}$$

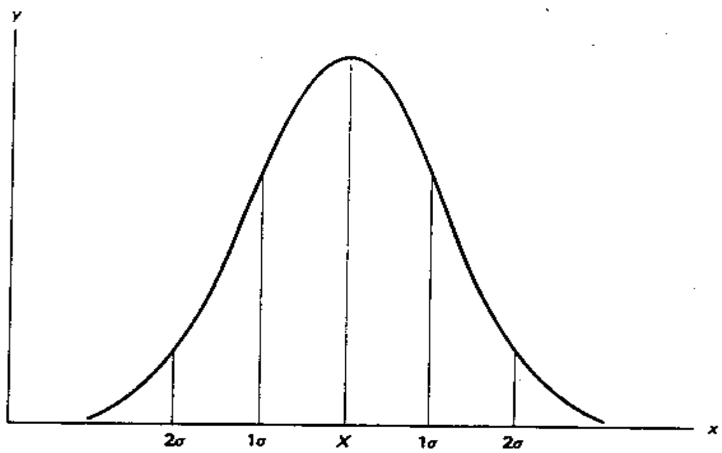
$$\sigma = \sqrt{\frac{\sum \left(x_i - \bar{x}\right)^2}{n-1}} \tag{2}$$

If you measure a quantity ***x*** an infinite, or very large, number of times, you would know the "true" distribution of that quantity, ***P*(*x*)**, the "true" mean and "true" standard deviation. There are subtleties in these obvious sounding claims that we will ignore, as the goal of this encounter with statistical thinking is to give you a familiarity with the key ideas. Your understanding of these concepts will deepen in time that you spend as a student of science.

In real life, you can never conduct an infinite number of trials. You may not want to make even a thousand measurements or a hundred: after all, it costs time and money whether you hire pollsters or scientists! One of the central tasks is to say as much as possible about ***P*(*x*)** from a small number of measurements of x_i, i= 1,2,...*n* (we call this a sample). The mean and standard deviation of this sample, of size ***n*,** is called the sample mean and sample standard deviation.

3. *Procedure*
3.1 Part I: Looking at Random Numbers

You will explore the fundamental idea of samples and universe, using Microsoft Excel as a tool. Actual distributions in life can be quite varied. Our software lets you play with a "theoretical" distribution called a Gaussian or Normal distribution: this is a popular choice among practitioners because of some elegant and useful aspects of this distribution. The normal distribution is displayed in Figure 1. It is useful to remember that the area enclosed within the interval $\bar{x} \pm \sigma$ for this Gaussian distribution is 68% and within the interval $\bar{x} \pm 2\sigma$ is 95%, within $\bar{x} \pm 3\sigma$ is 99%.



The relationship of 1σ and 2σ limits to the Gaussian distribution.

To construct an experimental histogram, consider the *range* of values that includes all of the trials under consideration. Next, divide that range into equally spaced "bins" and plot *how many* trials fall within each bin. That's a histogram. Your only real choice in creating a histogram is *how many* bins you divide the range into. Obviously, if you only divide the range into two bins the resulting histogram will not look very Gaussian no matter how many data points you've taken. <u>Typically, the number of bins should be chosen so that no more than 20% of your data falls within any one bin.</u> On the other hand, if you were to choose a very large number of bins, each bin interval would be very small; so you might have one or zero trials contained within each interval (undesirable).

- Open the file named "Random Error." As it opens, you will see the window shown in Figure 2

- Press the button labeled "Enable Macros."

- Save as a separate file before continuing.

- When the spreadsheet opens, you should see a spreadsheet with two buttons, as seen in Figure 3.

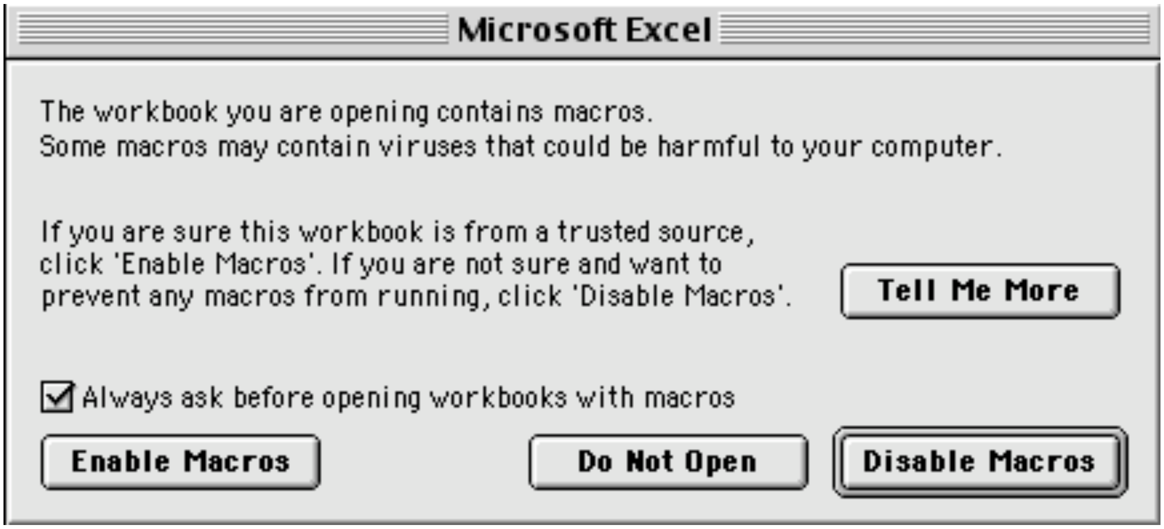


Figure 2

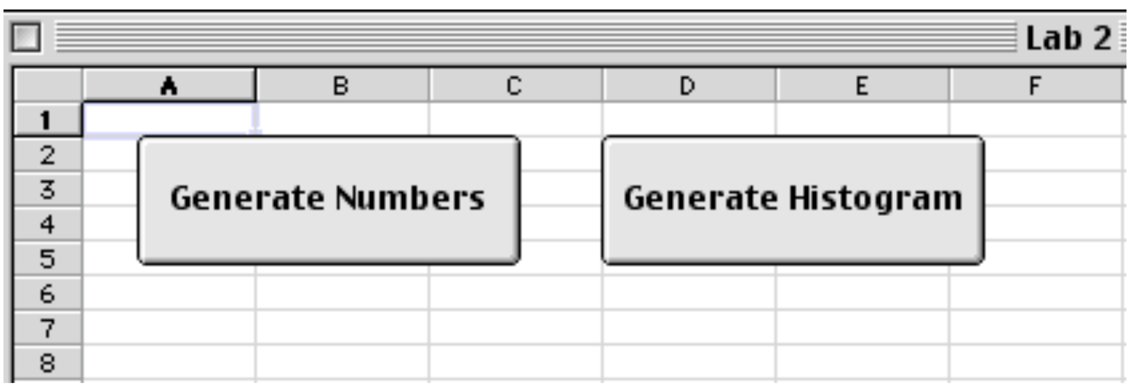


Figure 3

- To generate your data, press the "Generate Numbers" button. You should see the window shown in Figure 4. With this interface, you can tell the computer how many randomly generated numbers you want. The box labeled "Number of Samples" defines how many data sets you want. The box labeled "Data Points per Sample" defines how many data points you want in each set. The default values for the mean is **0.00,** the default value of standard deviation is **1.00**. You can play with this later. For now, leave it unchanged

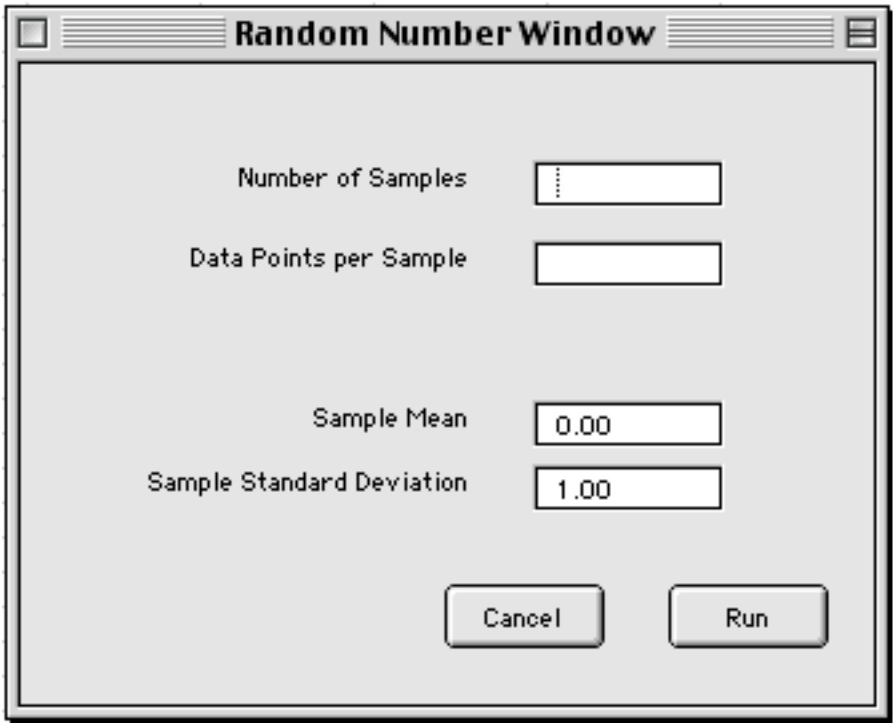


Figure 4

- ➢ To get a feel for how this works, begin by generating one data set with 2000 samples (*i.e.* enter "***1***" for the ***Number of Samples*** and 2000 for ***Data Points per Sample***). When you press the ***Run*** button, the computer *simulates* 2000 measurements of ***x*** and fills it in column A.

- ➢ Display your simulated 2000 measurements graphically by clicking the ***Generate Histogram*** button. You should see

  - the window shown in Figure 5.
  - the box labeled "Number of Bins" defines how many bins for the histogram.
  - the box labeled "Data Set" selects which set of data you want a histogram of, and
  - the box labeled "Histogram Name" allows you to name the worksheet that will hold the histogram.

Figure 5

- Let's create a histogram with 20 bins for the data set you just generated (*i.e.* enter "20" for the ***Number of Bins*** and "1" for ***Data Set)***. You can use whatever name you would like for ***Histogram Name***. The only catch is that you can not create two histograms with the same name. When you press the ***Run*** button, the computer will (after a brief delay) create another worksheet with the name you entered in the box ***Histogram Name***.

Does the histogram resemble Figure 1? *If* you had the patience to simulate 200,000 samples, the distribution would more closely resemble Figure 1. Don't waste time now!

In the upper left hand corner of the histogram sheet, you will see some relevant information on the data set that you just created a histogram for. The "mean" and "standard deviation" should be close to the default values of 0.00 and 1.00, respectively.

- Print out this sheet and attach it to you lab notebook.

- After you have looked at the histogram, return to the worksheet titled "Random Numbers," by selecting the Tab at the lower left hand side of the workbook, as seen in Figure 6.

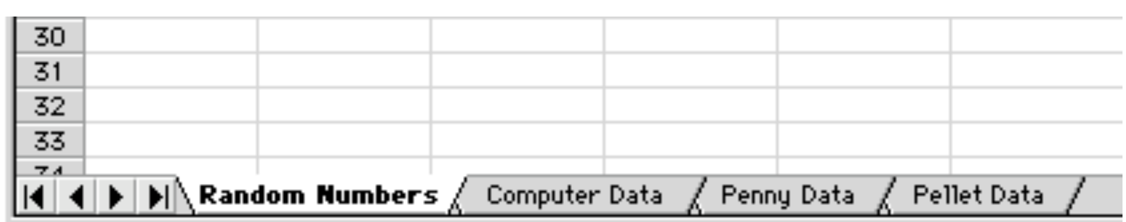


Figure 6

Now you are ready to explore the idea of small samples.

- Create five data sets, each containing 100 values.

- Display these samples graphically and inspect them. All five samples will appear different, though they may resemble each other. They may also resemble, but could be significantly different, from the normal distribution. In particular, the means could be significantly different from 0 and the histograms can appear very different from Figure 1, or "non-Gaussian."

    - For each data set copy and paste the mean into the sheet "Computer Data."
    - Do the same for the standard deviation.

- Print one "representative" sample, and attach it and make any relevant comments below.

- Complete the following table with the means and standard deviations of your five samples and the samples from your classmates.

Sample Number	Mean	Standard Deviation
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

- Enter your data into the sheet titled "Computer Data" and plot a histogram of these "sample means" and "standard deviations".
    - Click on the "Generate Histogram" button.
    - Enter an appropriate number for the ***Number of Bins***.
    - For "sample means", enter "2" for the ***Data Set***.
    - For "sample standard deviations", enter "3" for the ***Data Set***.

- Attach these graphs to your notebook and comment on them. Think carefully about, and *discuss,* the interpretation of these histograms.

3.2 Part II: The Mass of Pennies

- Select the sheet titled "Penny Data" and measure the mass of 40 pennies using the digital single pan balance.
- Enter the readings straight into the computer and create a histogram by pressing the ***Generate Histogram*** button. You will want to enter "2" for the ***Data Set***.
- Print the histogram and attach it to the notebook.
- Also, print the data table (*you'll need it*!) and attach it to the lab book.

- Compute the following:

mean = ________g σ = ________g (σ / mean) x 100 =

σ_m = g (σ_m / mean) x 100 =

3.3 Part III: Aluminum Pellets

- Select the sheet titled "Pellet Data" and measure the mass, diameter, and thickness of 35 aluminum pellets using the digital single pan balance and vernier calipers.
- Compute the volume of each pellet and enter the readings into the computer.
- Print a copy of the data table (*you'll need it*!) and attach it to the lab book.

- **Mass Data:** Create a histogram for the mass data by pressing the ***Generate Histogram*** button. You will want to enter "2" for the ***Data Set***. Print the histogram and attach it to the notebook.

- Compute the following:

mean = ________g σ = ________g (σ / mean) x 100 =

σ_m = ________g (σ_m / mean) x 100 = ________

- **Volume Data:** Create a histogram for the volume data by pressing the ***Generate Histogram*** button. You will want to enter "5" for the ***Data Set***. Print the histogram and attach it to the notebook.

- Compute the following:

mean = ________cm^3 σ = ________cm^3 (σ / mean) x 100 =

σ_m = ________cm^3 (σ_m / mean) x 100 = ________

4. *Questions:*

4.1 Part I: Random Numbers

i. You were to compare your early 2000 point sample, which approximates the Gaussian distribution fairly well, to results based on smaller data sets. It can be shown that the standard deviation of the mean σ_m and the standard deviation of the standard deviation σ_σ are related to the universe standard σ and the sample size n by the following relationships.

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_\sigma = \frac{\sigma}{\sqrt{2(n-1)}}$$

Can you see any way to use your class's data to comment on this expectation? Are your results consistent with this expectation?

ii. Based upon your work so far, what can you say about the relationship between sample size and the precision in estimating the mean and standard deviation distributions?

4.2 Part II: Pennies

i. What percentage of your sample falls within = ± σ of the mean? within ± 2σ of the mean? Do these results agree with your expectations?

ii. Clearly, the distribution for pennies is not Gaussian. Most of our sets of pennies produce distributions with two "most common" values, or two ***modes***. This kind of distribution is called a "bimodal" distribution. The deviations from the mean of this set are not like those encountered in the first part of this lab (*i.e.* Gaussian), so we expect that these variations are ***not*** due to random variation. If you encounter a non–random variation in your data, there is usually a reason (*i.e.* a physical mechanism) that causes the variation in question. List your guesses (verify if you can) for the main sources of the following variations.

(a) The bimodal nature for pennies?

(b) The distribution "within" each of the two "bumps"?

4.3 Part III: Aluminium Pellets

i. Why is the ratio (σ / mean) larger for volume measurement than for mass measurement?

ii. Try to think of or find somewhere a mathematical construct (other than mean and standard deviation) that can convey additional information about these distributions? Define it below and say in English what information it conveys.

5. *Initiative:*

i. Determine the measurement uncertainty for the pellet and penny data and discuss how you determined this value. Propagate these errors to determine the error in your calculated values.

5. *Conclusions:*