Name: ________________________

Partner: ________________________

Date: ________________________

Angular Momentum

1. Purpose:

In this lab, you will use the principle of conservation of angular momentum to measure the moment of inertia of various objects. Additionally, your aim is to try to develop a qualitative "feeling" for *rotational inertia*.

2. Theory:

For a system in the absence of any net *external* torque, the angular momentum, ***L***, is conserved (*i.e.* remains constant). This is the principle of conservation of angular momentum. In the case of two rotating bodies that undergo collision, conservation of angular momentum says:

$$L_{before} = L_{after}$$

$$\left(I_1\omega_1 + I_2\omega_2\right)_{before} = \left(I_1\omega_1 + I_2\omega_2\right)_{after} \tag{1}$$

3. Procedure:

In this experiment, you will examine a collision between an object having a *known* moment of inertia (a solid disk) and an object with an *unknown* moment of inertia. From the observed behavior, you can determine the moment of inertia of the unknown object using the principle of conservation of angular momentum.

- To do this, you will set a turntable (the disk) into rotation with an angular velocity ω_0, and hold the unknown object a ***very small*** distance above it. If we call the turntable "object #1", and the unknown "object #2", the angular momentum before the collision is:

$$L_{before} = I_1\omega_0 \tag{2}$$

- If the unknown object is suddenly dropped onto the rotating turntable so that the two objects rotate with a common angular velocity ω_f then:

$$L_{after} = \left(I_1 + I_2\right)\omega_f \tag{3}$$

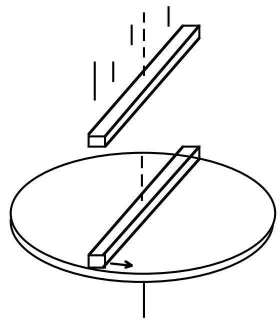

Figure 1: Depiction of a rotational collision

Applying the principle of conservation of angular momentum to this scenario yields Equation (4).

$$I_1\omega_0 = \left(I_1 + I_2\right)\omega_f \tag{4}$$

This can be solved for the unknown's moment of inertia *about the axis of rotation*:

$$I_2 = I_1\left(\frac{\omega_1 - \omega_2}{\omega_2}\right) \tag{5}$$

- You can find the moment of inertia of the turntable from static measurements and the formula from your text for the moment of inertial for a disk.
- Then, by measuring the initial and final angular velocities of the rotating objects, you'll be able to find a value for the moment of inertia of the unknown object.

This method can be used to determine the moment of inertia for any object and is, consequently, very useful when the geometry of the object is complicated enough that it would make an analytical calculation difficult. However, our goal here is more introductory, and so in this experiment, you will use three very *simple* objects as unknowns: a slab, hollow cylinder and a dumbbell. So, you will eventually be in position to compare your experimental results with theory, …but please *don't* look up the moments of inertia of these objects in your textbook just yet. We think it will be more helpful for you to try to make *estimates*:

- *Before* you start (and *before* you consult your textbook), try to *estimate* a value for the moments of inertia for the objects used in this lab.

Object	**Estimate** $\left[kg \cdot m^2\right]$
Slab	
Dumbbell	
Hollow Cylinder	

Later on, when you compare your initial estimate to your measurement, this exercise should help you to *retain* some sense of numerical magnitude for moments of inertia.

- As described above, you will set a turntable into rotation with an angular velocity ω_0, and hold the object with an unknown moment of inertia a ***very small*** distance above the turntable. Once the rotational speed of the turntable has been measured ***at least*** **twice**, you will drop the object.

Note: After collision, the dropped object and turntable must be at rest with respect to each other; otherwise the value you measure for ω_2 is not the same as in Equation (3).

- In order to compare your moment of inertia results to those presented in your textbook, you need to drop the unknown object onto the very ***center*** of the turntable. If it is dropped *off-center*, the moment of inertia you measure is still the moment about the axis of rotation - but is *not* the moment about any axis of symmetry of the unknown object itself, which is what is presented in your text. Your results will depend upon how well you align and drop the objects so that they are centered over the axis of rotation. It's up to you to find a good technique for doing this. [Does height perspective help or hurt?] Ask for any equipment you might need (plumb bobs, levels, *etc*.). A page of circular graph paper might be useful (perhaps you'd cut a hole in it?). You might wish to have each person in the group try dropping the objects five times without making any measurements, just to see who's best at dropping the objects into alignment. [After making your best effort, can the "**Parallel Axis Theorem**" help you to improve your analysis?]

- To determine experimental values for angular velocity, once again use the Data Studio™ software and a photogate using the "Photogate" sensor. — Attach a paper flag to the turntable. The photogate can measure the *time* it takes the paper flag to pass through a photogate and from this, if you have entered the width of the flag, you can calculate the average angular velocity of the disk during its passage through the photogate.

◉ What factors should influence your choice for the size of the paper flag that you will be using in this experiment?

◉ The flag passes through the photogate a distance R' from the axis of rotation. The width of the flag at this point is w. Use trigonometry to find the angle, $\Delta\theta$, subtended by the flag at the point it passes through the photogate:

- All of the data that you will require for your analysis should be recorded in clearly labelled tables (with ***units*** included).
- From your analysis, you should be able to calculate experimental values for the rotational inertial of the slab, of the dumbbell, and of the hollow cylinder. In each case, you should then consult your text, for formulas allowing you to analytically calculate theoretical predictions for the rotational inertia of each of these objects. ***Compare*** your experimental results with your theoretical predictions.

4. Questions:

Comment on the observed moment of inertia, both in comparison to theory and in comparison your initial estimates.

Are the collisions in this experiment *elastic* or *inelastic*? Explain carefully, using both equations and sentences.

Explain how it is possible for two objects with the same total mass and same radius to have very different moments of inertia.

In this experiment we assumed that no net external torque acted on our system; yet, if you set the disk rotating it will eventually decelerate and come to rest. Try to determine where this decelerating torque comes from and what its **numerical value** is. A separate experiment may be required to accomplish this. Describe the design of your experiment, any results you obtain and conclusions.

5. *Initiative:*

Possible ideas:

For those working in our labs at IWU, I wonder if it would be interesting to drop an IOLab device (from a ***very small*** distance) onto your spinning turntable? (If you are worried about your IOLab device flying off and crashing to the floor, remember that you can move your turntable to the floor, to it wouldn't fall far, if it did fly off.)

Alternatively, try activities on Pivot Interactives:

- Rotational Collisions: Disk on Disk (Momentum and Energy)
- Rotational Collision: Slab Dropped on Disk
- Rotational Collision: Dart Collides with Wooden Stick

6. ***Conclusions:***