Gabe Spalding's notes on a recent lecture by Kater Murch

Kater Murch is a relatively new professor of Physics at Wash U. He began his talk on "**The Arrow of Time in Quantum Measurement**" with a nod to his own undergraduate years, which were not at Wash U, but instead at Reed College, an undergraduate-only liberal arts college much like Illinois Wesleyan. – His first slide:

Prof. Murch's Question: Do you know who this image represents?
His answer: This is a fitting "mascot" for our chat today – Janus, the Roman god of beginnings and endings, shown with one face looking to the future and one to the past.

Second slide:

This image immediately tells you a story of before and after, where the past is distinct from what we have moving forward, and there's no going back

Third slide: Where the Second Law of Thermodynamics applies, we can speak of the tendency for entropy to increase as demarking a key distinction between the forward and backward directions in time. There are many ways for a child's bedroom to be messy, but few ways for it to be neat and tidy, so even if it is ever (momentarily) neat, it will quickly evolve towards mess.

On the other hand, the Second Law is a law of averages, as it were, while the *microscale* laws (both Newton's laws and those of quantum physics) are time symmetric: in other words, everything still "works" if you replace t with $(-t)$, and so $\vec{p}$ with $(-\vec{p})$.

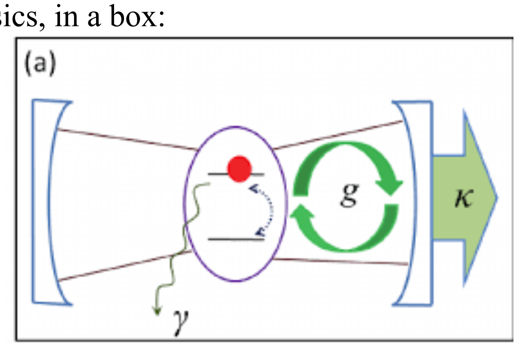
Andrew N. Jordon is a theorist at Rochester University who, lacking any lab of his own, simply made a few videos using a nearby (billiards) pool table. In the first of Andrew's slides, you see a single billiard ball moving from left to right and, next to that, the same video with the ball moving in the opposite direction. One of these is "correct," and the other is being played "backwards," but both look "acceptable." (If you look very closely and notice that in one video the ball appears to be slowing down, you still don't know whether that's because it is being played forwards and displays evidence of friction or because it is being played backwards and displays evidence of a *tilt* to the table surface.) His next slide shows side-by-side videos of a head-on collision of two balls (where, again, one of these is "correct," and the other is being played "backwards," but both look "acceptable."). After that comes a slide of a three-body interaction, followed – at last – by the first case where it is "clear" which is the "correct" video, where we see a cue ball "break" of the ordered, triangular array of balls that begins a game. Even there, however, it should be noted that the reversed video, where a seemingly random set of trajectories comes together in such a way that the cue ball is violently ejected, leaving the rest motionless, does NOT violate the rules of mechanics!

Prof. Murch stated that he collaborates with Andrew Jordon, on work which asks you, essentially, to look deep inside the pool balls, at the microscale, where we must explicitly consider the rules of quantum physics. Here, too, we find a time-reversible equation:

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi.$$

In quantum physics, what you do is to prepare a system, and then let Schrödinger's equation tell you how it will evolve in time …but, once you make a *measurement*, you have introduced some change that is analogous to the breaking glass in the second slide: some people speak of "wavefunction collapse" as a way of describing that you've broken symmetry. Another phrasing might involve the fact that we have extracted information from the system. (If you learn something, can you un-learn it?) In any case, measurement seems to introduce an asymmetry into the problem.

In Prof. Murch's lab, he makes use of a dilution refrigerator that cools down to temperatures that are as low as 10 mK above absolute zero. Such an apparatus is a bit like a Russian set of nested dolls, with each layer reaching a lower temperature. At the coldest layer, he includes a quantum device: an electrical circuit with two accessible states. He keeps this two-level system, as seems appropriate in quantum physics, in a box:



Here are the **rules** that summarize his experimental observations of the system he has prepared:

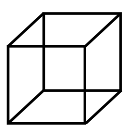
He can send in a pulse of the “right” amplitude, length, and frequency that will, with essentially 100% probability stimulate a transition (a “unitary” transition.) – either moving the “ball” in our cartoon from the top “shelf” to the bottom, if it started at the top (via stimulated emission of a photon), or moving it from the bottom shelf to the top, if it started at the bottom (via absorption of a photon).

So, what if he sends in “half” a pulse (characterized by half of the amplitude or a duration lasting half of the length of time of the one that yields a unitary transition)? – He says that he has done experiments with such pulses, yielding a 50% probability of a transition; that is, *half* of the time it “works.”

Next, what if he sends in a sequence of two half-pulses? – You might try to analyze this in terms of the two levels and predict that there are four equally likely outcomes, but that is ***NOT*** what he sees experimentally! Instead, he sees a 100% probability of transition, almost as if there were a kind of memory in the system, …but he ascribes this to **superposition**, which he described *as if* it were a virtual shelf somewhere between the top and bottom shelves. [Obviously, there are some constraints worth noting here: if the time interval between the two half pulses is a microsecond then the above statements hold. If that interval is 10 microseconds, then it drops to a 90% probability, *etc*. In other words, the system is characterized by a *decoherence* time.] He can also test the system with “quarter pulses,” and there are infinitely many possible superpositions:

$$\psi = \alpha|top\rangle + \beta|bottom\rangle$$

As a visual analogy, for the concept of superposition, consider the simple line drawing below:

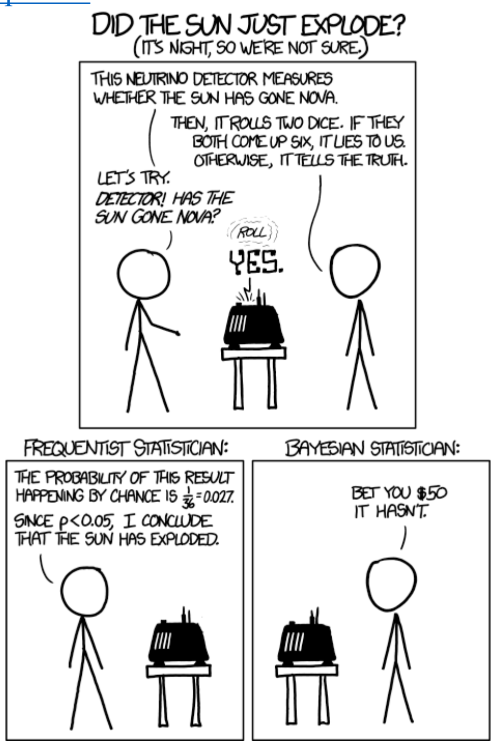


Is this a drawing of a box going (down and to the left), or one that goes (up and to the right)? Yes. (It is both.)

Schrödinger’s Cat was introduced to emphasize the paradoxical nature of the *measurement problem* of quantum physics: how does our system go from being in a superposition, to being “collapsed” into a particular state? Is it even possible to do experiments that probe the supposed “boundary”? Well, the sorts of experiments that Prof. Murch uses to try to explore this supposed boundary involve what are called “weak measurements,” which you might, in some sense, take to be akin to “rather gently” opening the box. In particular, what he does is to attenuate a coherent laser beam to the point where it sends a small number of photons into the box, through a small hole, aimed towards a small hole on the other side that leads to a detector. The first important detail is that by using an attenuated *coherent* laser beam as his source of photons, he is assured that they will arrive *randomly* (as “shot noise”): the fluctuations are key!!!

If the experiment is done many times, he can produce histograms representing:

P(N|bottom) = probability for # of photons detected given the “ball” is on the “bottom shelf,” &
P(N|top) = probability for # of photons detected given that the “ball” is on the “top shelf.”
For example, these might look like overlapping gaussian distributions, where the most probable number of photons detected when the system is in the ground state is three, while the most probable number of photons detected when the system is in the excited state is five. Then, when the system is prepared so as to be in a superposition, let’s suppose you do a measurement and you detect seven photons. That might make you believe that the “ball is most likely on the top shelf,” but here Prof. Murch suggests a quick reading of Randall Munroe’s web comic *xkcd* entitled, “[Did the Sun Just Explode?](#)”



The Bayseian analysis uses Bayes’ Theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

In the case of the comic, the right-hand side multiplies the probability that the roll of the dice was *not* double sixes (*i.e.*, 1 - 0.027 = 0.973) by the probability that the sun has gone supernova (essentially zero), yielding that the left-hand side must also be essentially zero.

In the case of the weak measurements done in Prof. Murch’s lab, even if you work it all out using quantum operators, you end up with essentially the same result as you would expect from Bayseian analysis:

$$P(top|N) = \frac{P(N|top)P(top)}{P(N)}$$

$$P(bottom|N) = \frac{P(N|bottom)P(bottom)}{P(N)}$$

Experimentally, an important step is to update our state of knowledge, as we continue measuring.

The result is a plot which he calls a "quantum trajectory," showing the probability that our "ball" is on the top shelf, as a function of time. Importantly, these quantum trajectories reflect some amount of *reversibility*, consistent with the theoretical work of A. N. Korotkov, A. N. Jordan, *PRL* **97** (16) 166805 (2006), and the experimental work (from the folks behind Google's quantum computing efforts) of N. Katz, *et al., PRL* **101** (20) 200401 (2008), and of Y.-S. Kim, *et al.*, *Nature Physics* **8**, 117-120 (2012). The fundamental question is this: what are the conditions for being able to "play the game backwards" (*i.e.*, for the system to exhibit **reversible** behavior)?

Let's, for a moment, return to our earlier consideration of the cue ball "break" that opens a game of billiards. The analogy for our weak measurement experiments is that we want to play the same guessing game as to the *likelihood* of the time-reversed process, by continually updating a *total* path probability, $P_{tot} = P_1 \times P_2 \times P_3 \times P_4 \cdots$, as we continue to detect photons, so as to end up with a "forward probability" and a "reverse probability." – Even as we find that quantum trajectories can be reversible, there emerges, over time, a clear asymmetry between the forward and reverse processes, as reported in the work of J. Dressel, *et al.*, *PRL* **119**, 220507 (2017). – If we make 100000 measurements, then experiment reveals a *slight* preference to the forward processes over the reverse processes, but if you continue running the experiment, the asymmetry grows, in just the fashion predicted by the "Fluctuation Theorem." Prof. Murch claims that these experiments are revealing how the Second Law of Thermodynamics *emerges*, and in doing so establishes the Fluctuation Theorem as an extension of that law.

Summary:

1) Classical Physics is time reversible, but there emerges an arrow of time that is associated with entropy (which often acts as a tendency towards increased disorder) and the Second Law of Thermodynamics.
2) Quantum Physics is time reversible, except when measurement occurs, and this gave rise to explorations of "**weak measurement**," which is also time reversible, and allows us to see how the Second Law emerges.
3) A notable difference between the cases of Classical Physics and Quantum lies in the fact that, in Quantum Physics, the transition to irreversible behavior (associated with a measurement) corresponds, in some sense, to a transition from a more "complex" state (*i.e.*, superposition) to a simpler ("collapsed") state. [Gabe interjects that measurement of the microworld involves **amplification**, which corresponds, in some sense, to a transition from a simple degree of freedom to a complex ensemble of degrees of freedom!!!]

The Future: ("Quantum Revolution 2.0")

The first quantum revolution led to the development of silicon transistors and lasers and so on, but in the 21st century, we are entering a second quantum revolution, where the quantum "weirdness" (of superposition and entanglement, which we haven't discussed today) isn't just *underlying* device development, but "front and center." – There is an inherent complexity to quantum mechanics: a modest array of qubits requires more parameters to model than the number of particles that exist within the observable universe. The coming quantum revolution will allow us the opportunity to harness that capital, even without us ever being able to know that capital, fully. We are now at the cusp of a moment being dubbed as "Quantum Supremacy," when it will no longer be possible to classically mimic quantum processors. Already, we have begun to explore squeezed light and other forms of "Quantum-

Enhanced Metrology;" *e.g.*, when LIGO next upgrades, they will be exploiting entanglement to even further reduce their noise floor, by yet another order of magnitude! That factor of ten extends their reach in each direction, meaning that they will be sampling a volume of space that is 1000 times larger, yielding 1000 times more detected events. With that, we expect they will be observing black hole mergers every day. – With enhanced metrologies, it may finally become possible to sort out the connection between quantum and gravity. For all of these (many) opportunities, one key is to clarify the connection to "information" and to "entropy."

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Q&A:
As an undergraduate, one of Prof. Murch's teachers at Reed College was the theorist David Griffiths (author of the most ubiquitous advanced undergraduate texts on Quantum Mechanics, on Electrodynamics, and on Particle Physics), who would often point out that a birthday cake where all of the icing is lumped up in one corner has lower entropy than one where the icing is smoothly distributed across the surface. [Gabe interjects that Sean Carroll, a theorist from Cal Tech, has a series of five "Minute Physics" videos where he discusses the distinction between entropy and complexity.]

Q: In 1874, Josef Loschmidt criticized Ludwig Boltzmann, over the issue of time reversibility. Can you please place this talk into that historical context?

A: Here, we are examining small systems, where the fluctuations are large, and we find results that are consistent with the Fluctuation Theorem:

$$\frac{P(+\Delta S)}{P(-\Delta S)} = e^{\Delta S}$$

In the limits discussed, this should be expressed as an inequality ("The Jarzynski Inequality"). [Gabe interjects that interested students can use weak optical traps to explore this in our labs, here at Illinois Wesleyan.]

Q: You described the distributions you obtain, for the number of photons counted, as gaussian distributions, but photon counting for a coherent state should give Poisson statistics, right?

A: What's actually being measured here is phase (or a phase *shift*), which does give gaussian statistics. You might imagine that it is *as if* the "ball" had an index of refraction. In any case, it is phase uncertainty which leads to weak measurement, here.

Q1: When you make a measurement, you gain information. Is there the potential to extract work from this?

(related) Q2: Landauer's Principle holds that writing or erasing a bit of information always comes at a cost of at least $k_B T ln(2)$. Does that principle still hold in the quantum limit?

A: Qubits are different than classical bits, because they can exist in a superposition of states. So, they must be described by a *revised* Landauer bound, which is derived by replacing the "Shannon Entropy" with the "Von Neumann Entropy." Nevertheless, in the end, you arrive at the same sort of result.

Q: What would be the result if your source were not an attenuated coherent laser, but were instead a photon source that prepared states that are eigenfunctions of the number operator (*i.e.*, a "Fock State")?

A: The result of using a Fock State is that you would have what we call a "projected measurement," where there would be no quantum uncertainty. However, all of our results discussed here today are entirely contingent upon the use of weak measurement with small signal-to-noise ratio due to the inclusion of large quantum fluctuations.

Further reading:

See the Murch group website, and begin by scrolling to the bottom, where there are popularizations of his work, created by the press. – If, after that, you would like more background, you should begin with his book chapter/review article. His email is murch@physics.wustl.edu